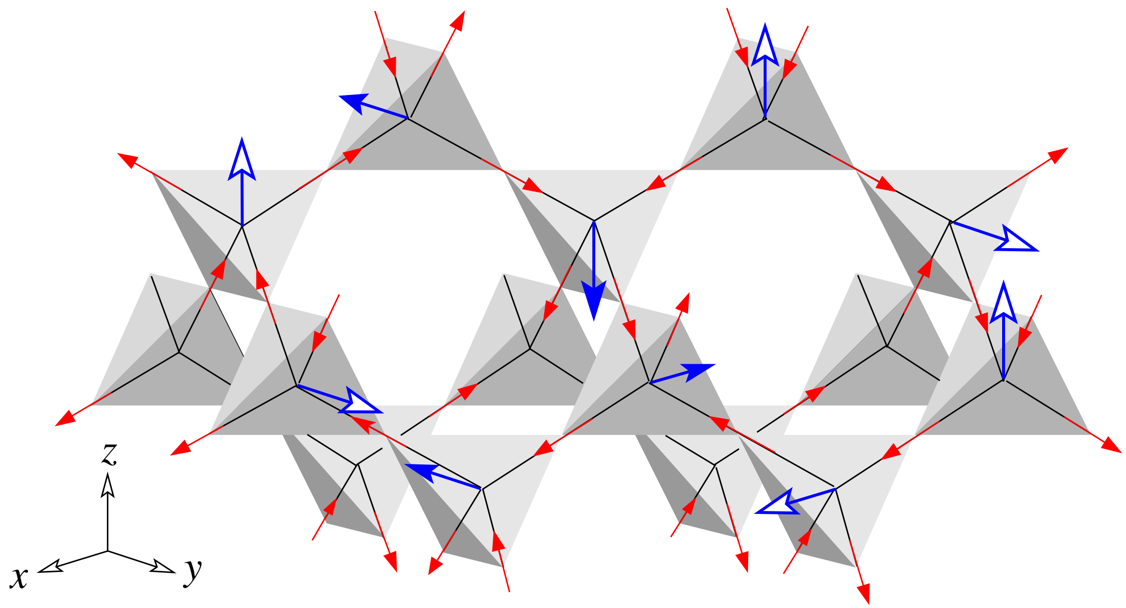
Orbital Ice: p-band Mott insulator on the diamond lattice



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NGSCES Santiago de Compostela 2011

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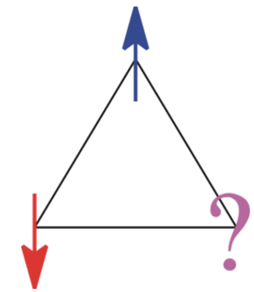
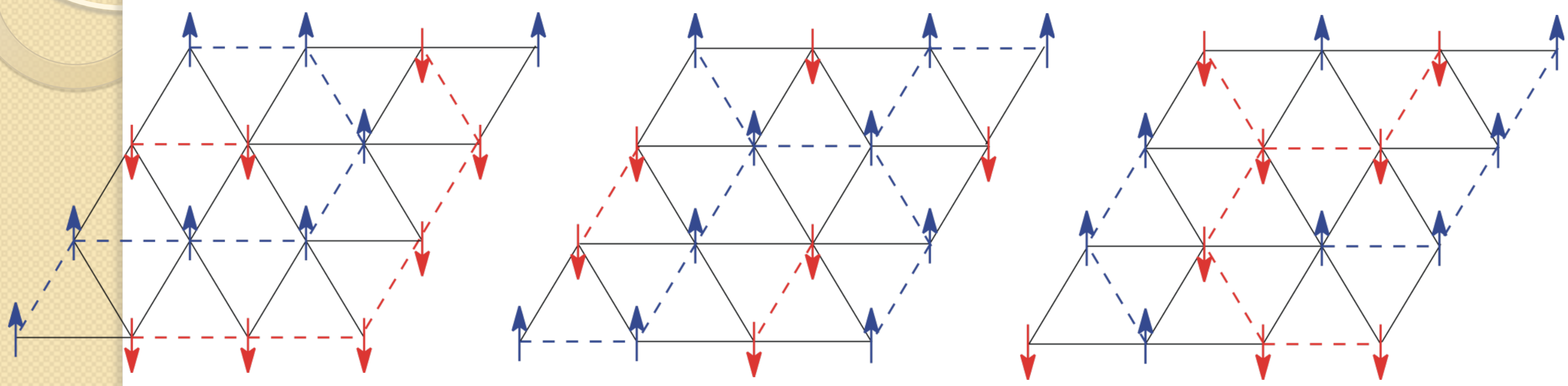
Outline

- Geometrical frustration:
  - Macroscopic ground-state degeneracy.
  - Perturbations $\rightarrow$ rich and diverse magnetic phases.
- Spin ice:
  - Fractionalized excitations: magnetic monopoles.
  - Emergent gauge structure and coulomb phase.
- Orbital ice:
  - p-band Mott insulators in optical diamond-lattice.
  - Exact many-body ground states forming a ice manifold.
- Conclusion and outlook.

Geometrical frustration



Triangular Ising antiferromagnet



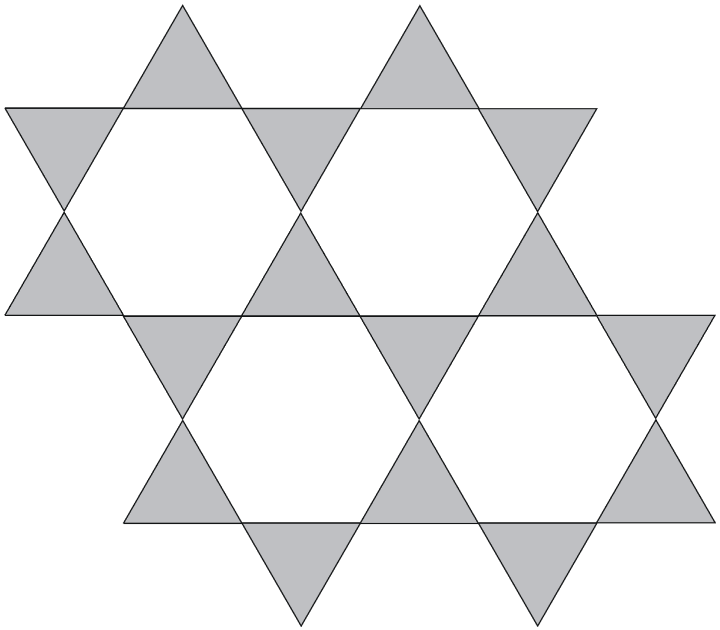
- One frustrated bond on each $\triangle$.
- # of ground states $W_{\mathrm{gs}} = e^{\mathrm{const}\times N}$

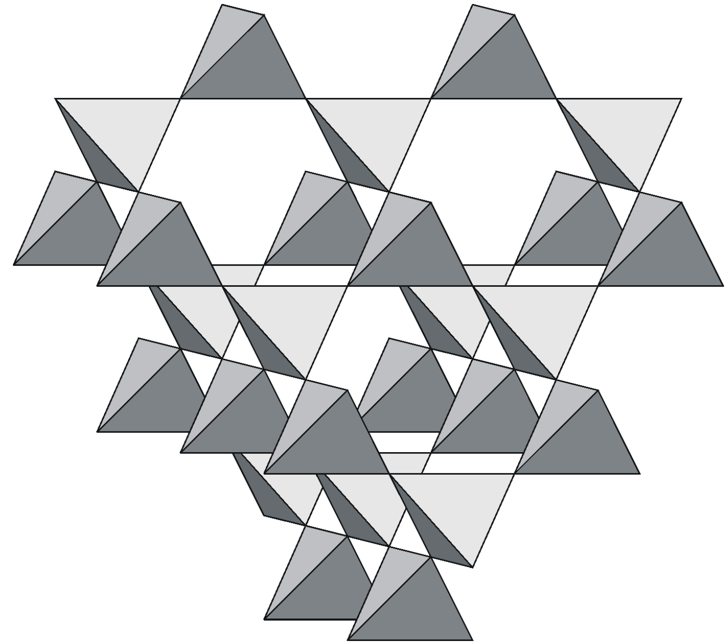
$$\mathrm{const} = \frac{2}{\pi}\int_0^{\pi/3} \ln(2\cos x)\, dx = 0.323066$$

- Residual entropy $S = k_B \ln W_{\mathrm{gs}} \propto N$.
- No long-range spin order down to $T \to 0$.
  power-law spin correlation $\langle \mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_0 \rangle \sim \mathrm{const}/r^2$.

G. H. Wannier, R.M.F. Houtappel (1950)

Antiferromagnet on Bi-Simplex Lattice





$$\sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j = \frac{1}{2}\sum_{\triangle} |\mathbf{S}_{\triangle}|^2$$

$$\sum_{\langle ij\rangle} \mathbf{S}_i \cdot \mathbf{S}_j = \frac{1}{2}\sum_{\boxtimes} |\mathbf{S}_{\boxtimes}|^2$$

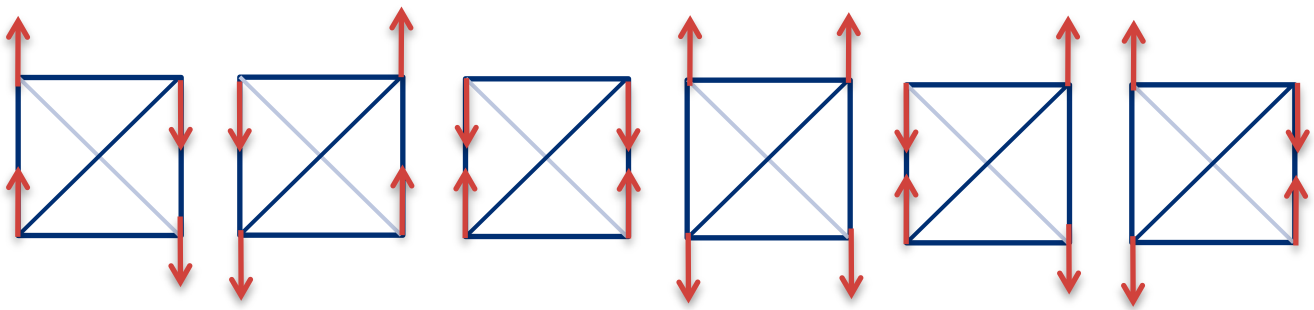
Ground states: total spin = 0 on every simplex

$$\mathbf{S}_{\triangle} = 0 \quad (\forall \triangle)$$

$$\mathbf{S}_{\boxtimes} = 0 \quad (\forall \boxtimes)$$

How many Ground States ?

Discrete Spins (e.g. Ising spin):



6 out of $2^4 = 16$ configurations satisfy $\mathbf{S}_{\boxtimes} = 0$, a fraction of $\frac{3}{8}$.

$$\text{Number of ground states: } W_{\text{gs}} \approx \left(\tfrac{3}{8}\right)^{N_{\text{tet}}} \times 2^N = \exp\left(\tfrac{N}{2}\ln\tfrac{3}{2}\right).$$

$[N_{\text{tet}} = N/2]$

$$\Longrightarrow \text{Residual entropy: } \tfrac{S}{N} = k_B \tfrac{1}{2}\ln\tfrac{3}{2} = 1.68 \text{ J/(mol K)}.$$

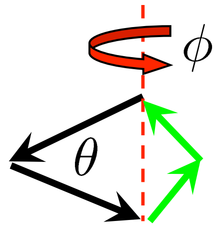
[Pauling's estimate]

Experimental spin-ice $Dy_2Ti_2O_7$: $S_0/N = 1.86$ J/(mol K).

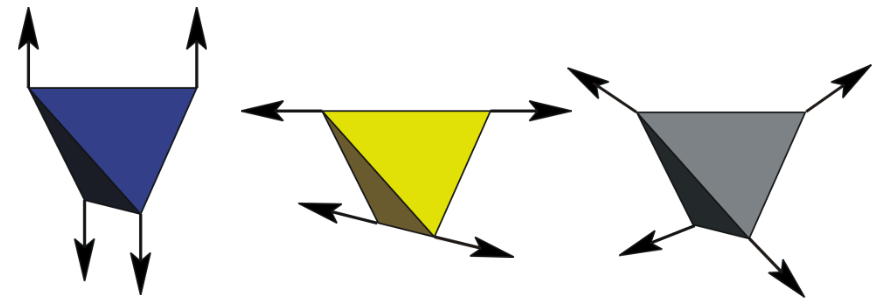
[Bramwell and Gingras (2001)]

Continuous (Heisenberg) Spins

$$\mathbf{S}_{\boxtimes} = 0$$



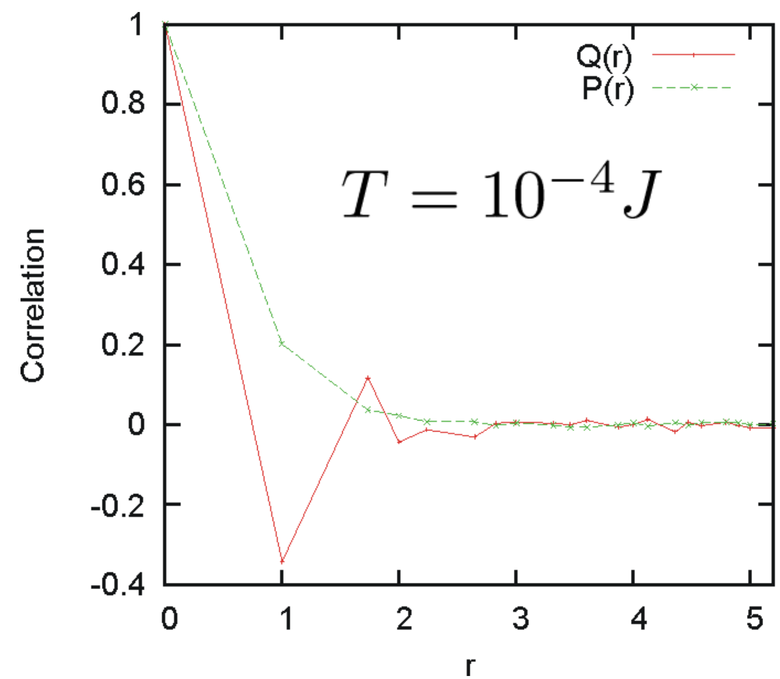
Ground states:
Rigid rotation $+(\theta, \phi)$



Pyrochlore Lattice:

The degenerate ground states form a continuous manifold
$\mathbf{x} = (x_1, x_2, \cdots, x_D)$

$$D = N_{\boxtimes} = N_{\mathrm{spin}}/2$$



Liquid-like spin correlation

Macroscopic degeneracy : Hypersensitivity to perturbations

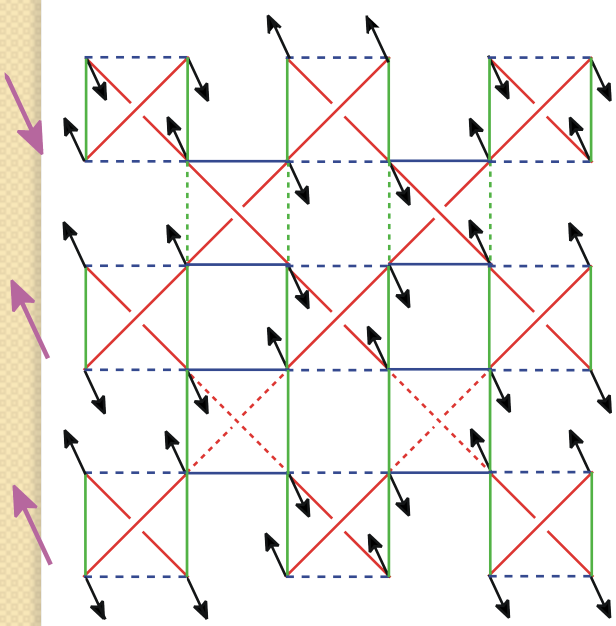
Relieving the frustration:

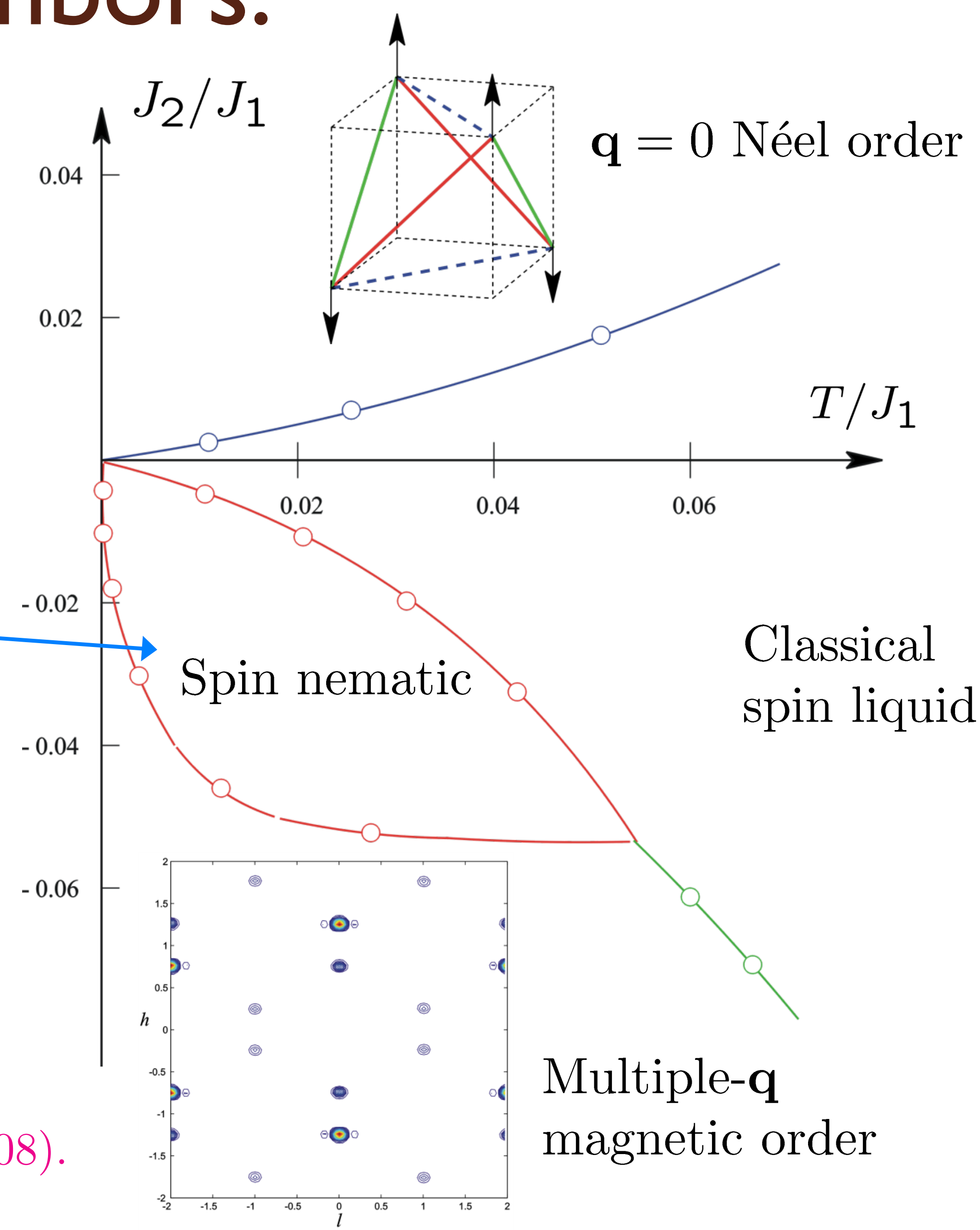
- Quantum fluctuations (order-by-disorder):
  Valence bond solid
- Further neighbor interactions:
  Néel order, spin nematic
- Long-range dipolar interaction:
  Spin ice ($Ho_2Ti_2O_7$ and $Dy_2Ti_2O_7$)
- Spin-Lattice coupling:
  Néel order ($ZnCr_2O_4$, $CdCr_2O_4$)
- itinerant electrons, double-exchange coupling:
  Non-coplanar magnetic order

$\cdots$

Further-neighbors:

J_1-J_2 Heisenberg spins on pyrochlore lattice





G.-W. Chern, R. Moessner, O. Tchernyshyov, PRB (2008).

Spin-Lattice coupling

- magnetoelastic coupling:

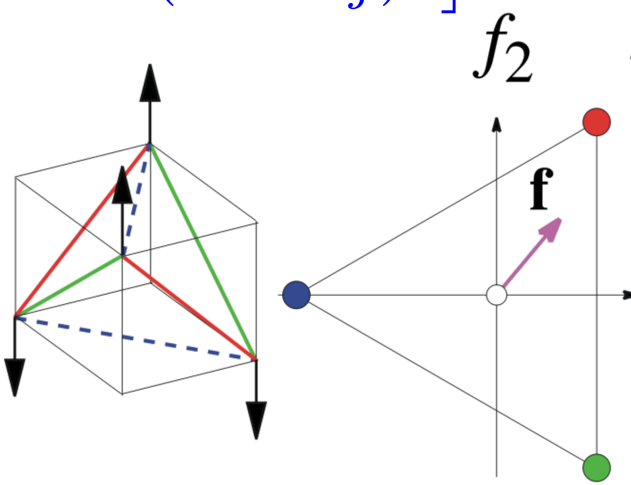
$$E_{ij} = J(r_{ij})\mathbf{S}_i \cdot \mathbf{S}_j = J_0 \mathbf{S}_i \cdot \mathbf{S}_j + J' \delta r_{ij} (\mathbf{S}_i \cdot \mathbf{S}_j).$$

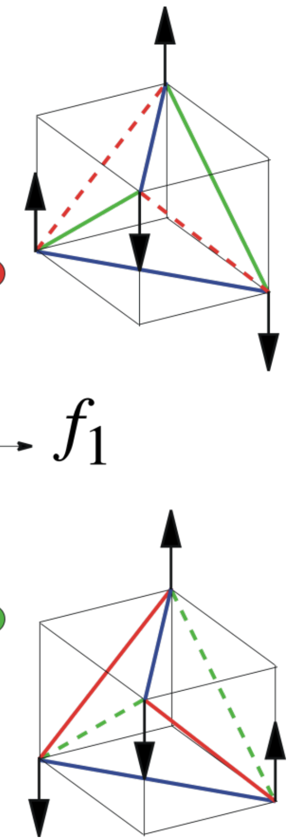
- elastic energy: $\frac{k}{2}(\delta r_{ij})^2$.

$$\Rightarrow E_{\text{eff}} = \sum_{\langle ij \rangle} \left[J \mathbf{S}_i \cdot \mathbf{S}_j - K (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \right]$$

- Collinear spins in ground state !

- Tetragonal flattening along x, y, and z.





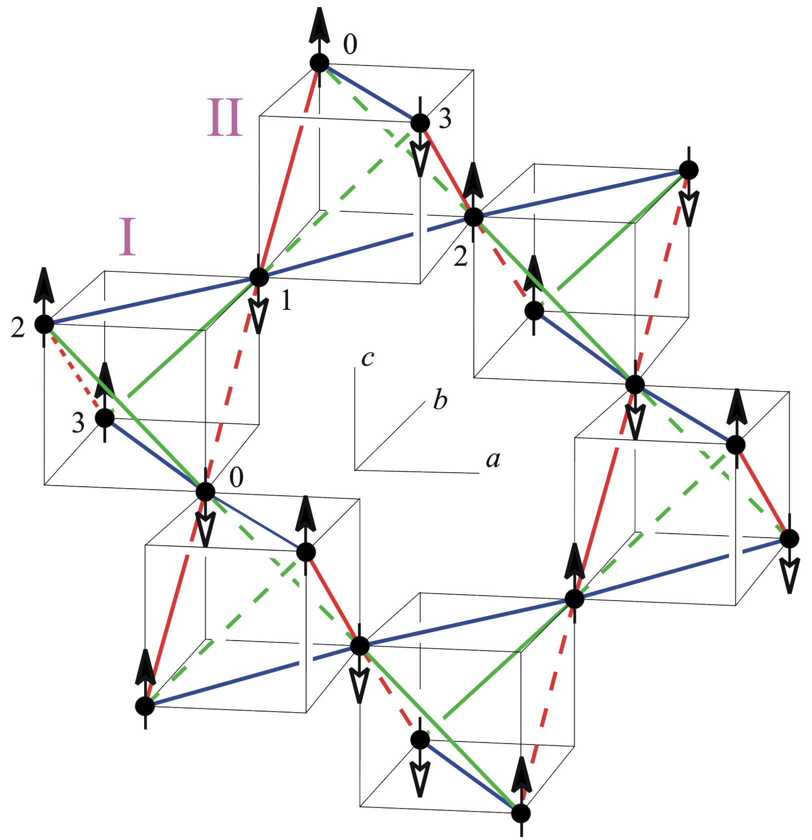
O. Tchernyshyov, R. Moessner, S. Sondhi, PRB (2002)
G.-W. Chern, C. Fennie, O. Tchernyshyov, PRB (2006)

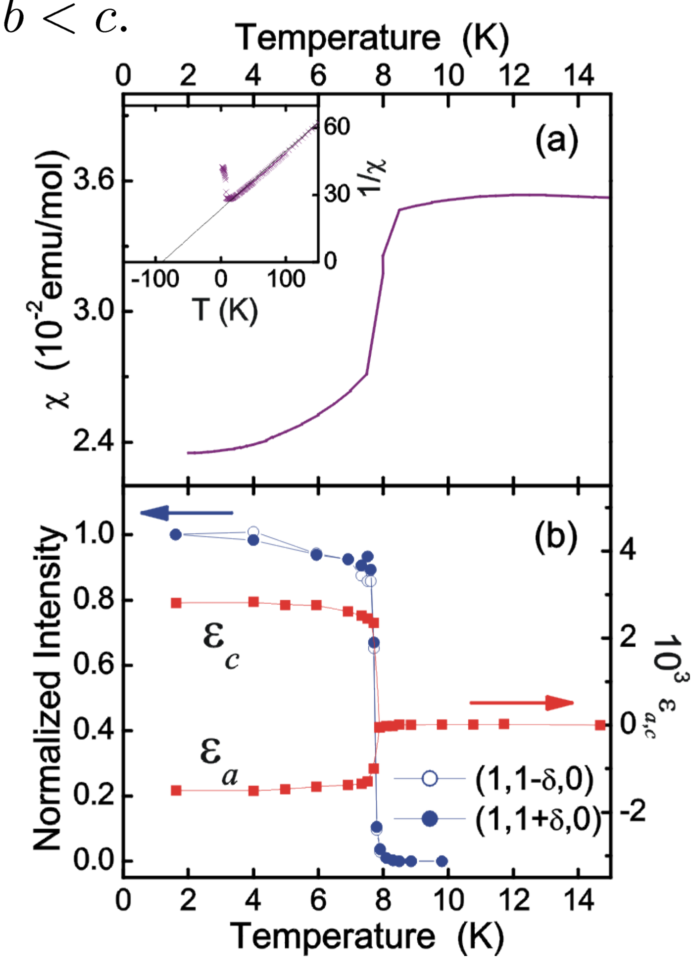
Ex: CdCr2O4

J.-H. Chung *et al.*, (2005).

- 1st-order transition at $T = 7.8$ K.
- Tetragonal distortion $a = b < c$.

Staggered distortion:





G.-W. Chern, C. Fennie, O. Tchernyshyov, PRB (2006)

Coupling to Itinerant electrons

- Double-exchange or Kondo-lattice model:

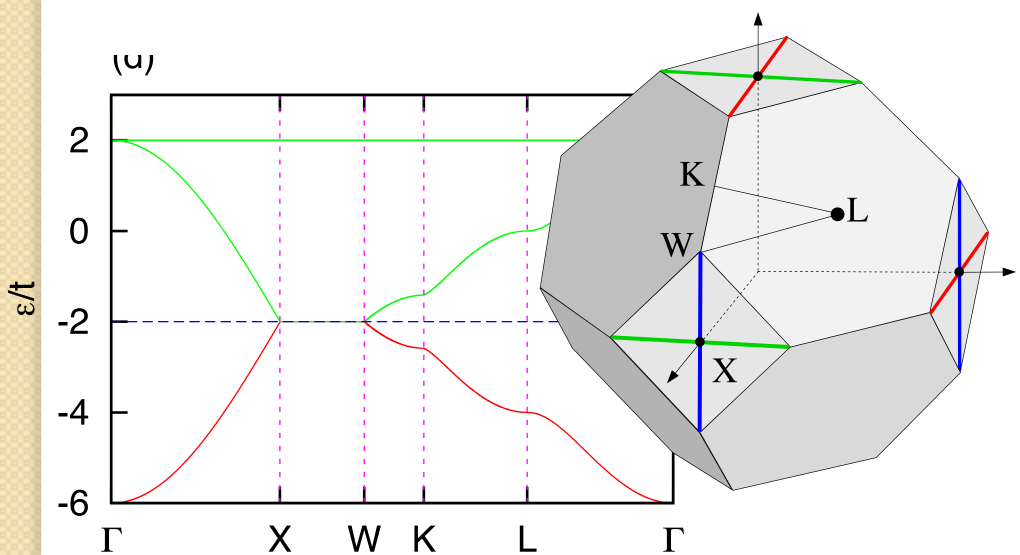
$$H = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

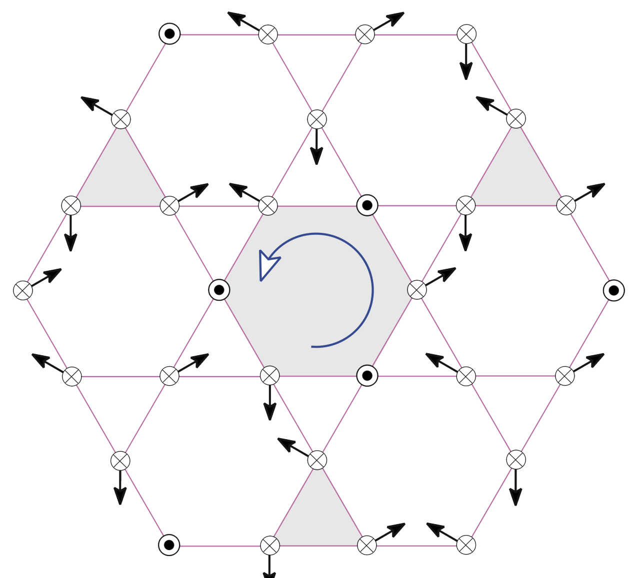
G.-W. Chern, PRL (2010)

$$+K \sum_i \mathbf{S}_i \cdot c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta} - t \sum_{\langle ij \rangle} \left(c^\dagger_{i\alpha} c_{j\alpha} + \text{h.c.} \right)$$

SU(2) invariant Hamiltonian. *No* spin anisotropy.

Chiral Non-coplanar magnetic order !

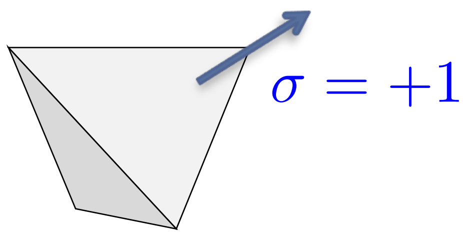


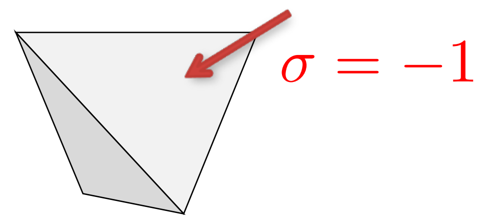


Spin Ice:

- Pyrochlore *ferromagnet* of rare-earth ions.
  (e.g. $Di_2Ti_2O_7$ with $S = 15/2$)

- Strong crystal-field anisotropy $\Delta_{\mathrm{CF}} \approx 200$ K along local [111] axis $\Rightarrow$ $\mathbf{S}_i \approx \sigma_i \, \mathbf{e}_i$ at $T \sim 10$ K, Classical Ising spins $\sigma_i = \pm 1$.





- a simple Spin-ice Hamiltonian:

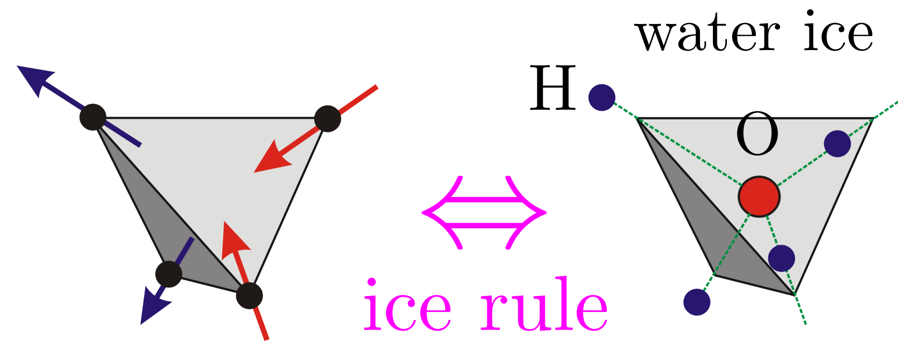
$$H = -J_F \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j = +\frac{J_F}{3} \sum_{\langle ij \rangle} \sigma_i \sigma_j = +\frac{J_F}{6} \sum_{\boxtimes} \sigma_{\boxtimes}^2 .$$

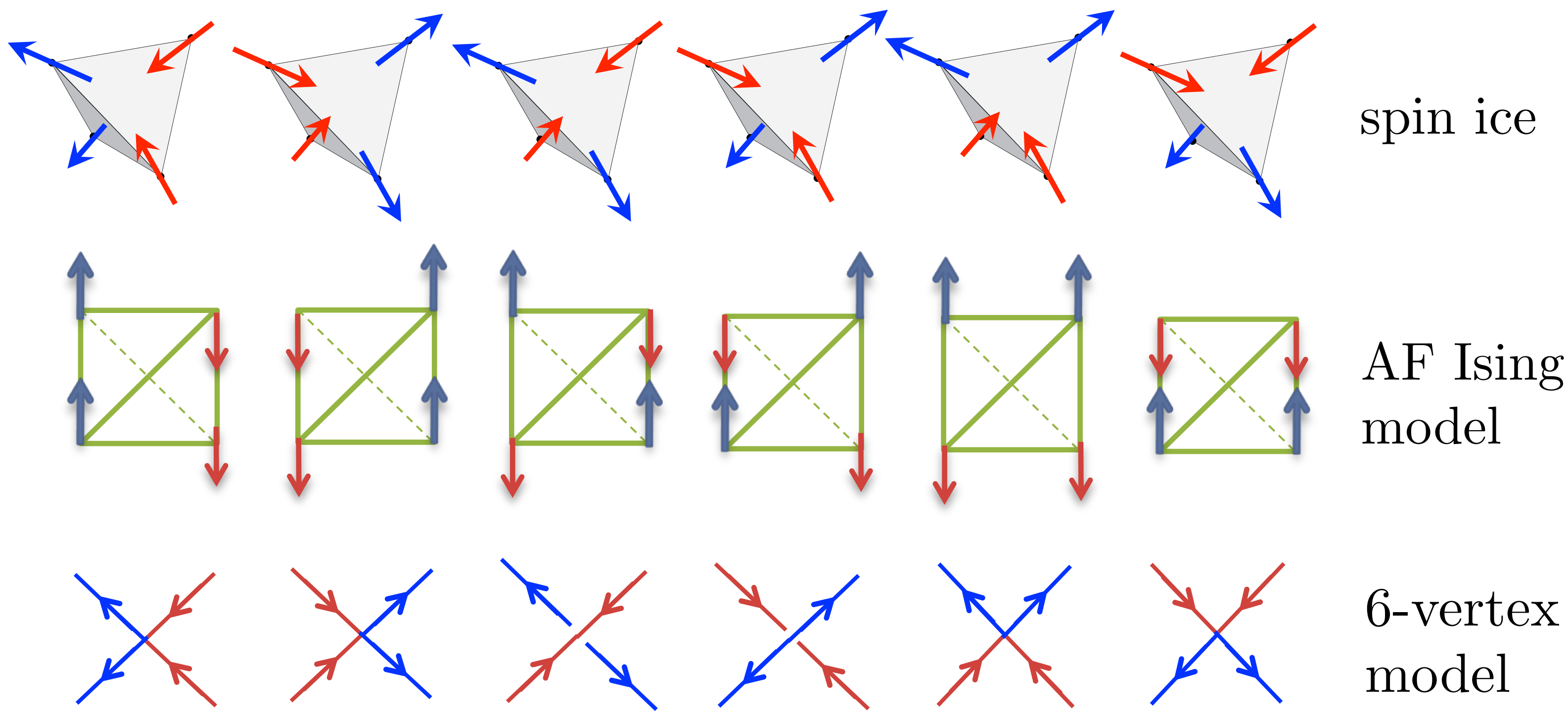
$\Rightarrow$ Ising antiferromagnet on the pyrochlore lattice

Ice Rule

- Local 2-in-2-out order:

$$\sigma_{\boxtimes} = \sum_{i \in \boxtimes} \sigma_i = 0$$

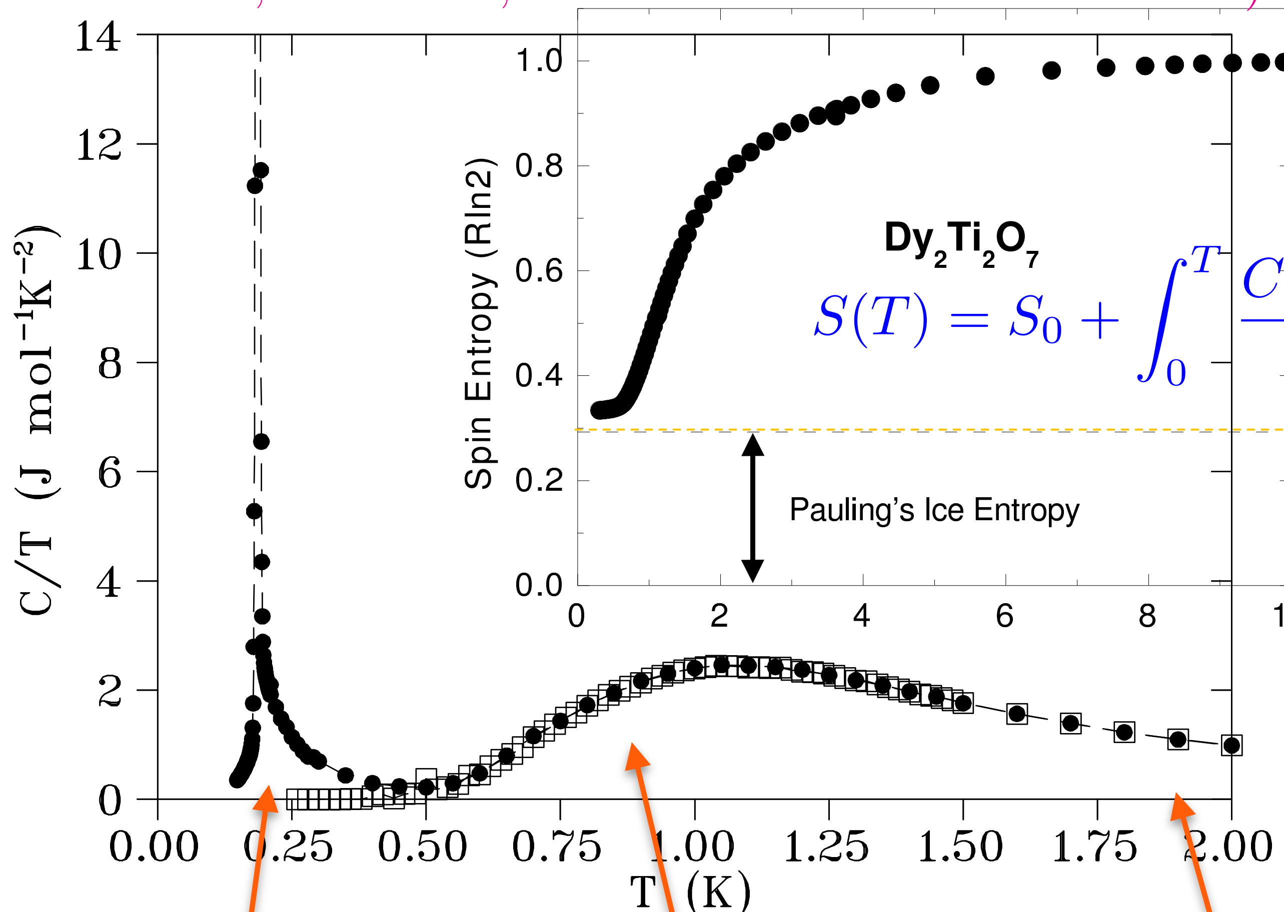




- Macroscopic degeneracy: $\dfrac{S_0}{N k_B} \approx \dfrac{1}{2} \ln \dfrac{3}{2}$

Evidence of ice-rule (residual entropy):

(R. Melko *et al.*, PRL 2001, A. P. Ramirez *et al.* Nature 1999)



$$S(T) = S_0 + \int_0^T \frac{C(T)}{T} dT$$

Magnetic ordering

onset of "ice rule" (2-in-2-out)

High-T paramagnet

Why 'dipolar' spin ice obeys ice rule ?

- A realistic Hamiltonian: $(J \approx 1 \sim 2$ K and $D \approx 1.4$ K$)$

$$H = \underbrace{\frac{J}{3}\sum_{\langle ij\rangle}\sigma_i\sigma_j}_{\substack{\text{exchange}\\ \text{interaction } E_{\text{ex}}}} + \underbrace{\frac{Da^3}{2}\sum_{ij}\sigma_i\sigma_j\left[\frac{\mathbf{e}_i\cdot\mathbf{e}_j}{r_{ij}^3} - \frac{3(\mathbf{e}_i\cdot\mathbf{r}_{ij})(\mathbf{e}_j\cdot\mathbf{r}_{ij})}{r_{ij}^5}\right]}_{\substack{\text{dipolar}\\ \text{interaction } E_{\text{dip}}}}$$

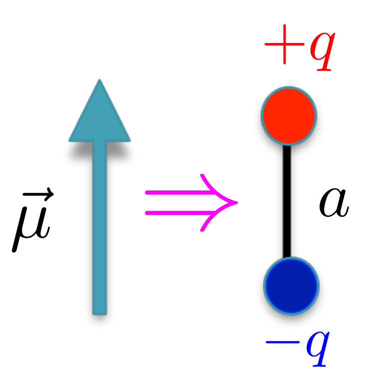
- For spins $\{\sigma_i\}$ satisfy ice rules: $\sum_{i\in\boxtimes}\sigma_i = 0$ for all $\boxtimes$.

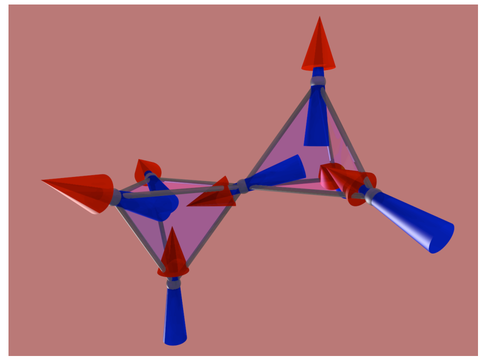
$$\Rightarrow E_{\text{ex}}(\{\sigma_i\}) = \text{const.}$$

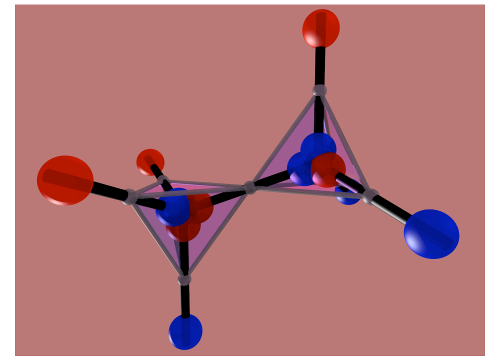
However, Why is the dipolar energy also almost the same, $E_{\text{dip}}(\{\sigma_i\}) \approx \text{const}$??

Dumbbell Picture

Why is the long-range dipolar interaction (almost) irrelevant ?







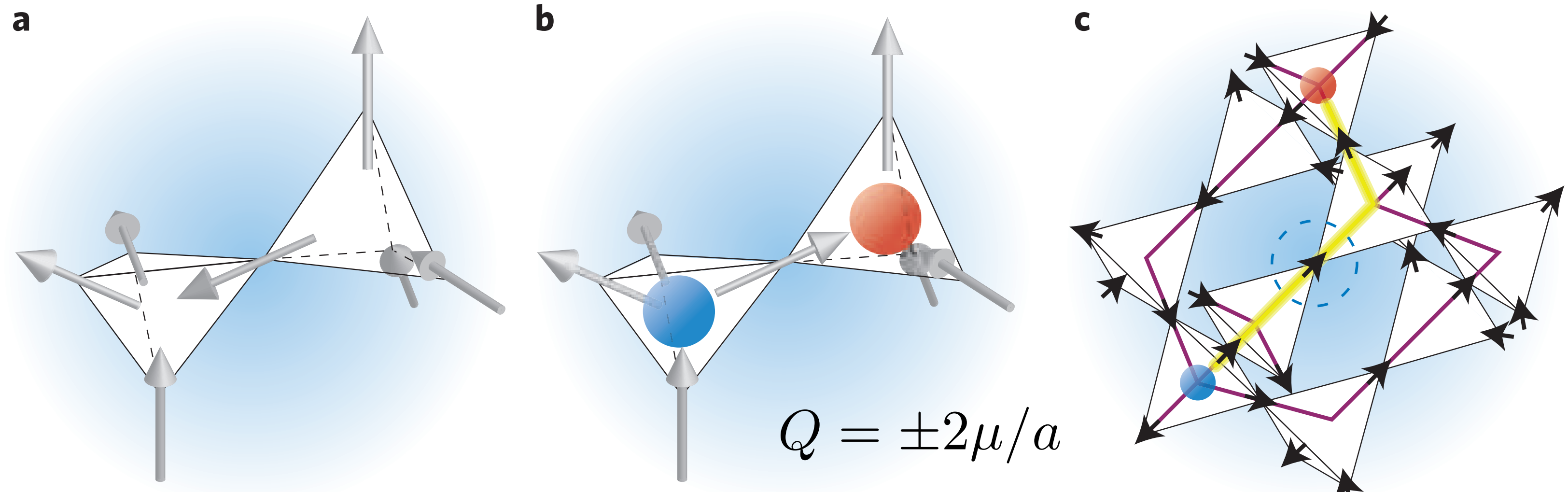
$$H \Rightarrow \sum_{\alpha} \frac{v_0}{2} Q_\alpha^2 + \frac{\mu_0}{8\pi} \sum_{\alpha \neq \beta} \frac{Q_\alpha Q_\beta}{|\mathbf{r}_\alpha - \mathbf{r}_\beta|} + H_{\text{quad}}$$

$$Q_\alpha = \sum_{i \in \alpha}^{\boxtimes} \sigma_i \ : \ \text{magnetic charge of a tetrahedron}$$

- Low-energy states: $Q_\alpha = 0 \Rightarrow$ 2-in-2-out ice rule !
- All microscopic ice states have (almost) the same energy.
- $H_{\text{quad}} \sim \mathcal{O}(1/r_{\alpha\beta}^5)$ responsible for the low-T magnetic ordering.

Fractionalization: Magnetic Monopoles

(C. Castelnovo *et al.* Nature 2008)
(Z. Nussinov *et al.* PRB 2006).

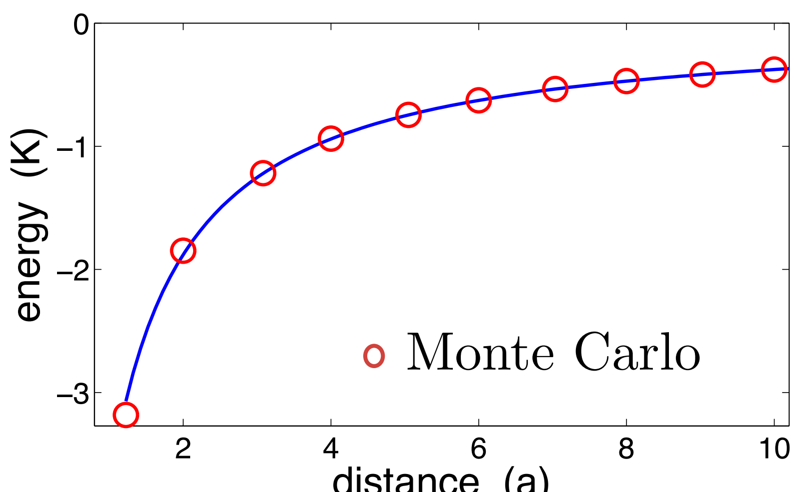


$Q = \pm 2\mu/a$

ice rule: $Q = 0$

a flipped dipole

two deconfined monopoles !



$$V(r) = -\frac{\mu_0}{4\pi}\frac{Q^2}{r}$$

3D Coulomb interaction

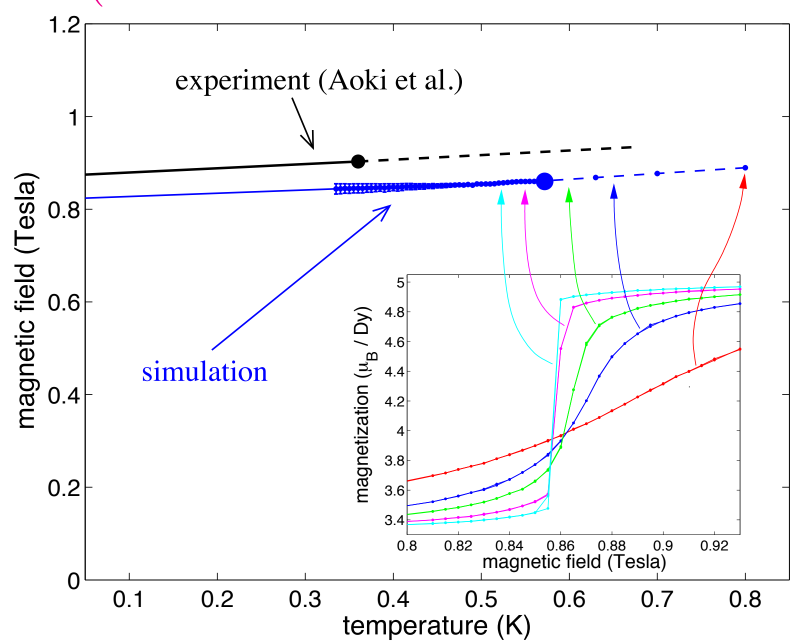
Observing monopoles

Dirac quantization:

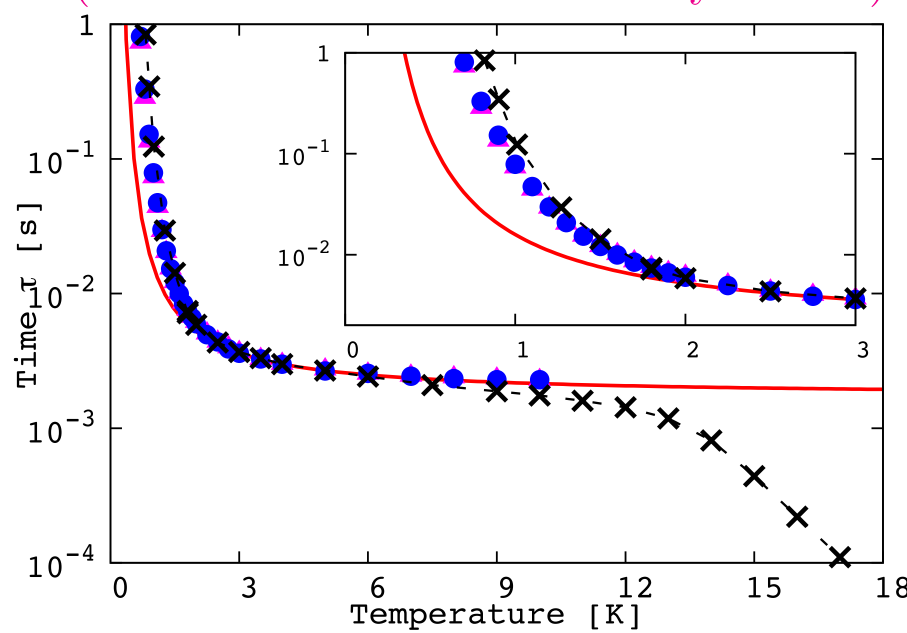
- monopole charge: $q_m = \frac{2\mu}{a} \approx \frac{q_D}{8000}$. $\left(eq_D = \frac{nh}{\mu_0},\ n = 0, 1, 2, \cdots\right)$

  $\Rightarrow$ Difficult (but not impossible) to observe directly.

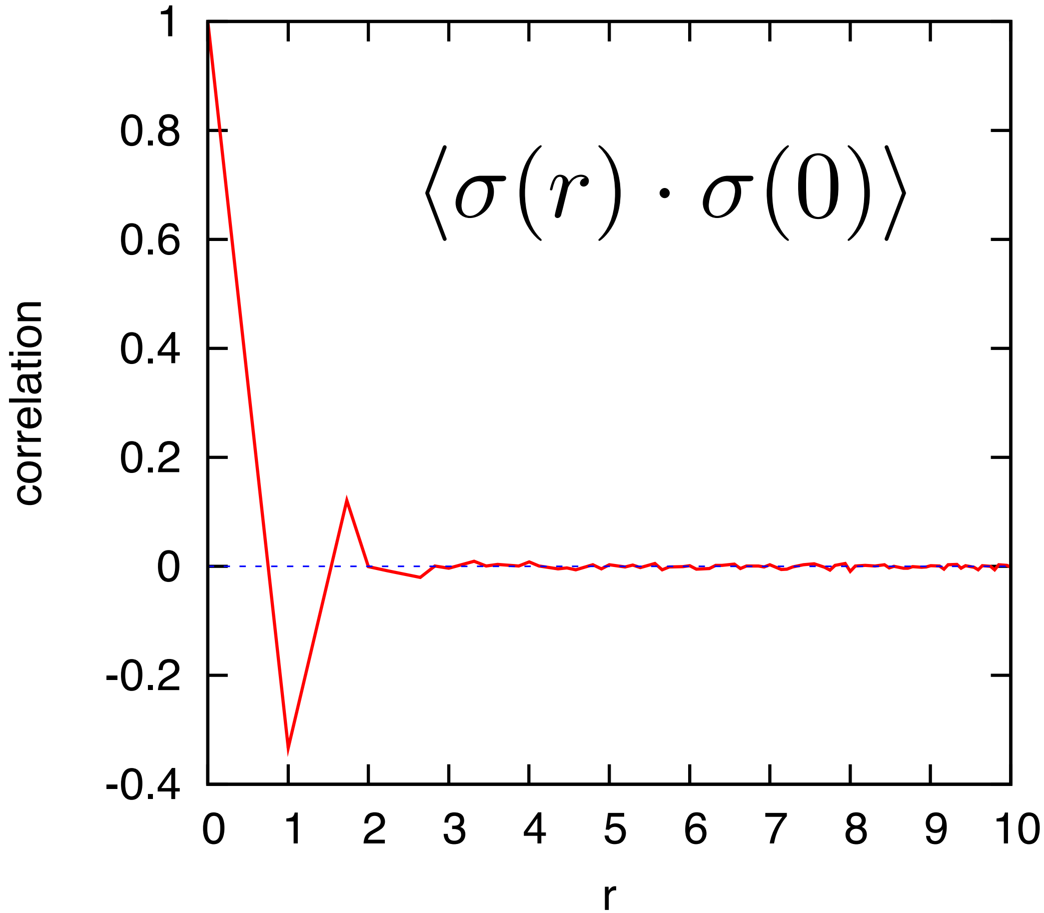
- Liquid-gas transition of monopoles:
  (C. Castelnovo *et al.* Nature 2008)



- Magnetic relaxation in spin ice:
  (L. Jaubert *et al.* Nature Phys. 2009)



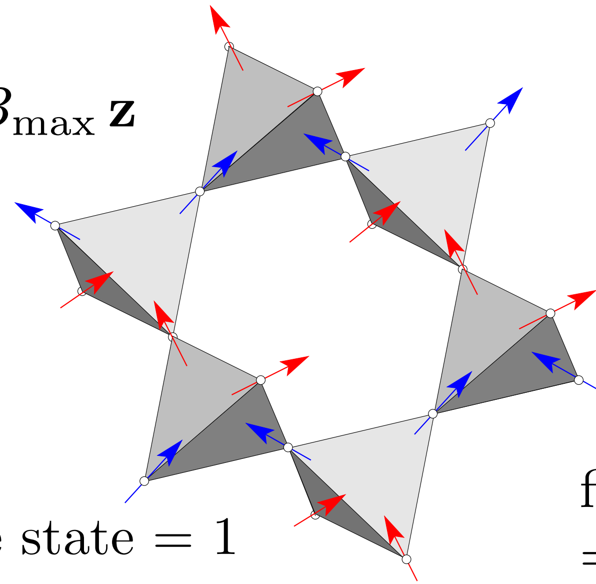
Is the ice phase really ‘featureless’ ?



Emergent Gauge structure

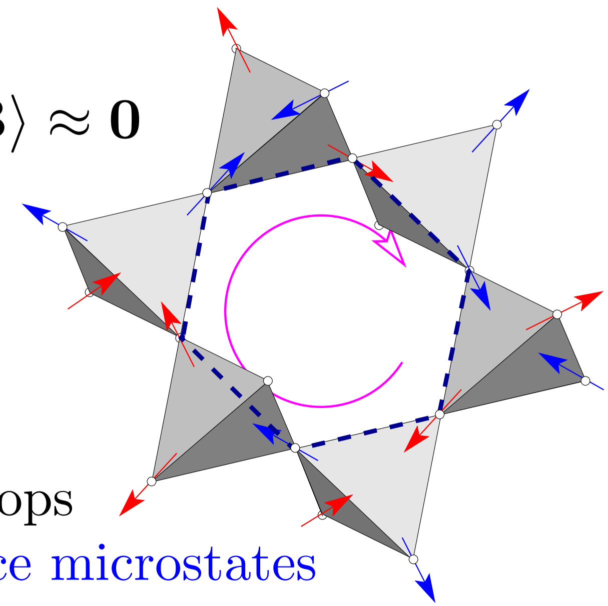
- Define a ‘magnetic’ field: $\mathbf{B}(\mathbf{r}_i) = \mathbf{S}_i = \sigma_i \mathbf{e}_{m(i)}$.

$\langle \mathbf{B} \rangle = B_{\max}\,\mathbf{z}$



\# of ice state = 1

$\langle \mathbf{B} \rangle \approx \mathbf{0}$



flippable loops

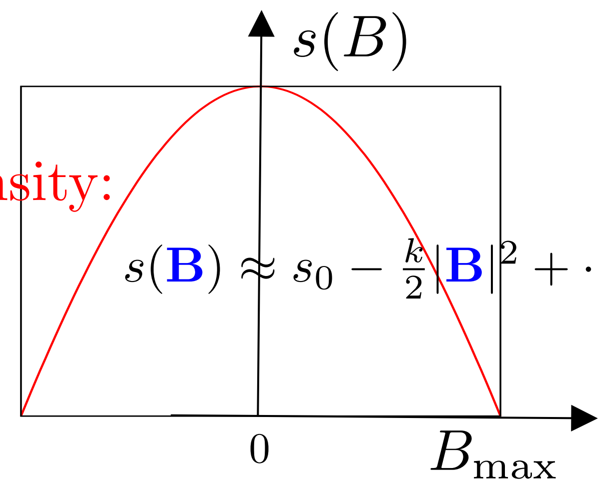
$\Rightarrow$ lots of ice microstates

$$Z = \sum_{\text{ice states}} \propto e^{-s(\mathbf{B})\times V}$$

$$\stackrel{?}{=} Z_0 \int \mathcal{D}\mathbf{B}(\mathbf{r}) e^{-\int d^3\mathbf{r}\,\frac{k}{2}|\mathbf{B}|^2}$$

entropy density:

$s(\mathbf{B}) \approx s_0 - \frac{k}{2}|\mathbf{B}|^2 + \cdots$



Emergent Coulomb phase

(Isakov *et al.* PRL 2004)
(C. Henley PRB 2004)

$$\text{Ice rule} \Rightarrow \sum_{i\in\boxtimes} \sigma_i = 0 \text{ (Ising spins)}$$

$$\Rightarrow \nabla\cdot\mathbf{B}(\mathbf{r}) = 0 \text{ (coared-grained approx)}$$

- emergent magnetostatics (Coulomb phase):

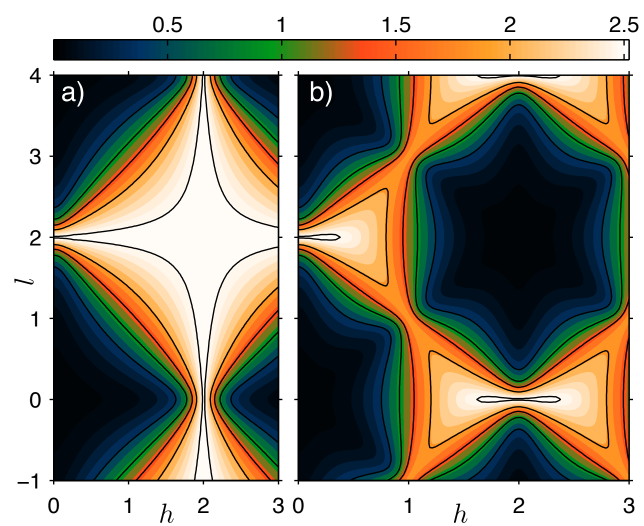
$$Z = Z_0 \int \mathcal{D}\mathbf{B}(\mathbf{r}) \prod_{\mathbf{r}} \delta\left(\nabla\cdot\mathbf{B}(\mathbf{r})\right) e^{-\int d^3\mathbf{r}\,\frac{k}{2}|\mathbf{B}(\mathbf{r})|^2}$$

$$= Z_0' \int \mathcal{D}\mathbf{A}(\mathbf{r}) e^{-\int d^3\mathbf{r}\,\frac{k}{2}|\nabla\times\mathbf{A}(\mathbf{r})|^2} \quad (\mathbf{B} = \nabla\times\mathbf{A})$$

- Dipolar correlation:

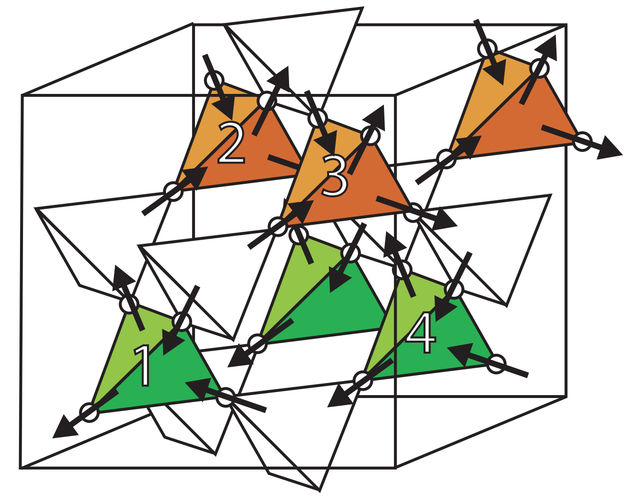
$$\langle B_i(\mathbf{r})\, B_j(0)\rangle \approx \frac{4\pi}{k}\frac{3x_ix_j - r^2\delta_{ij}}{r^5}.$$

$\Rightarrow$ Pinch-pint singularity in structure factor:

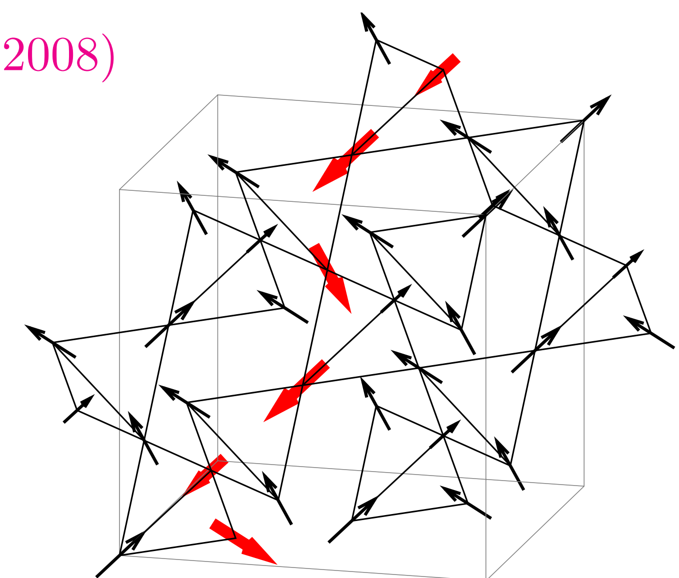


Transition out of Coulomb phase

- Residual quadrupolar interaction
  $\Rightarrow$ 1st-order transition to a $q = (001)$ magnetic order.
  (R. Melko *et al.* 2001)



- Coupling to electrons (RKKY):
  $\Rightarrow$ 1st-order transition to a $q = (001)$ magnetic order
  (A. Ikeda & H. Kawamura, 2008)



- Magnetic field $H \parallel [001]$
  $\Rightarrow$ Kasteleyn transition to a $q = 0$ magnetic order.
- Uniaxial pressure
  $\Rightarrow$ infinite-order transition to a $q = 0$ magnetic order.
  (L. Jaubert *et al.* 2008, 2010)

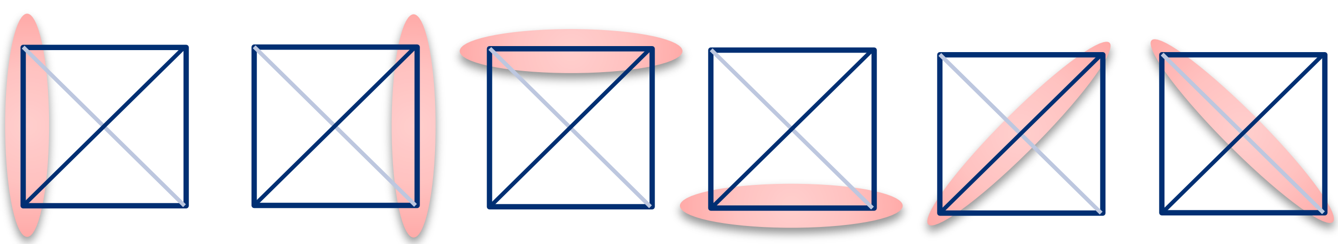
Other 'ice' models: Klein S-1/2 model

- Klein spin-1/2 model on pyrochlore lattice:

  (Z. Nussinov, C. D. Batista, B. Normand, & S.A. Trugman, PRB 2007)

$$H_K = J \sum_{\boxtimes} \mathcal{P}^{S_{\boxtimes}=2} + \cdots$$

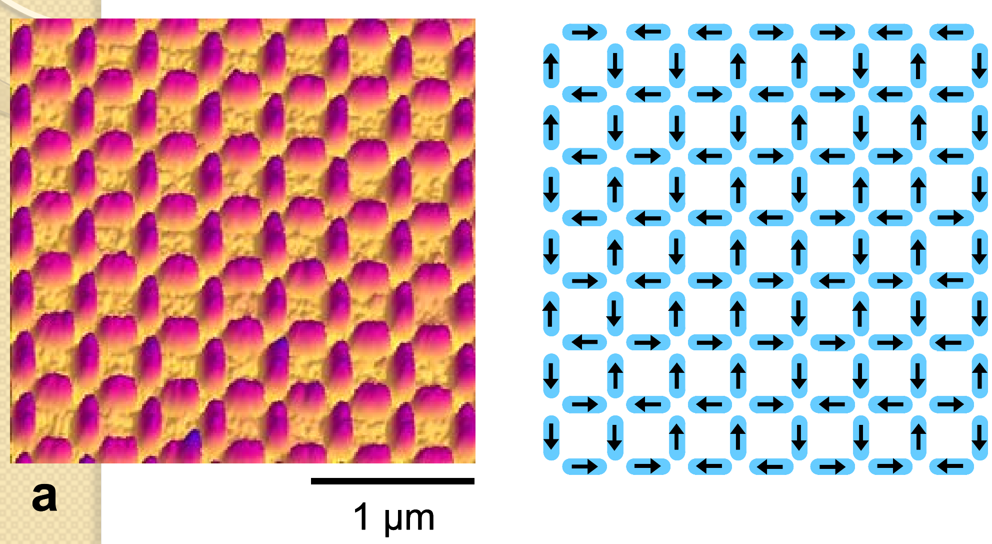
- Ice rules $\Rightarrow$ hard-core dimer covering:



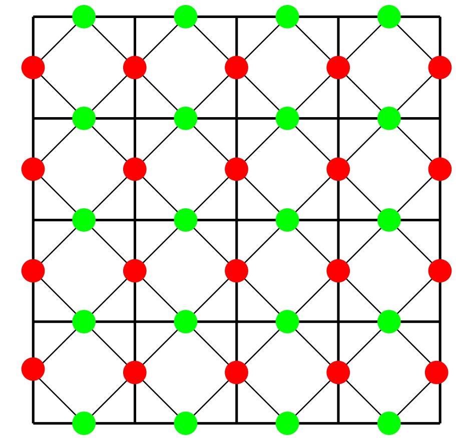
- Fractionalized excitations:
  Deconfined spinons (monopoles) in 3D.

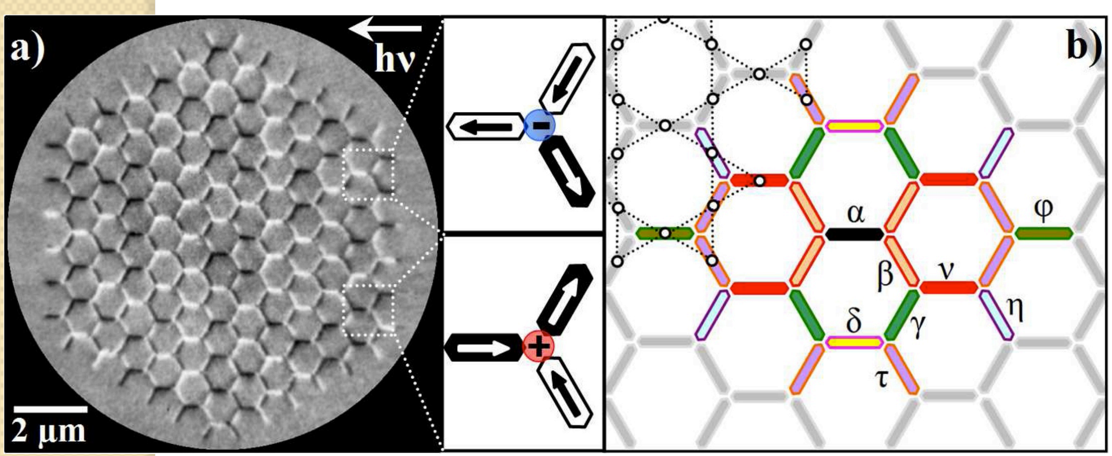
Other ‘ice’ systems: magnetic nano-arrays

- Artificial spin ice in 2D square and kagome lattices:

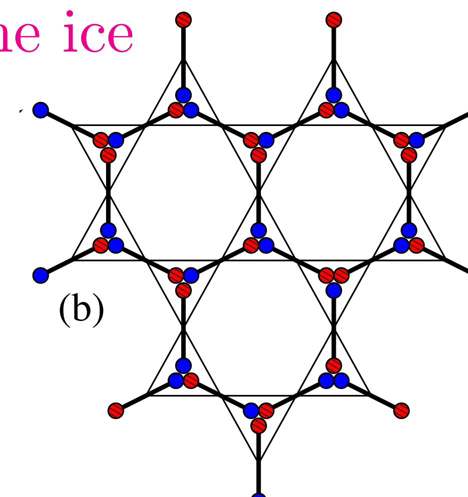


checkerboard ice





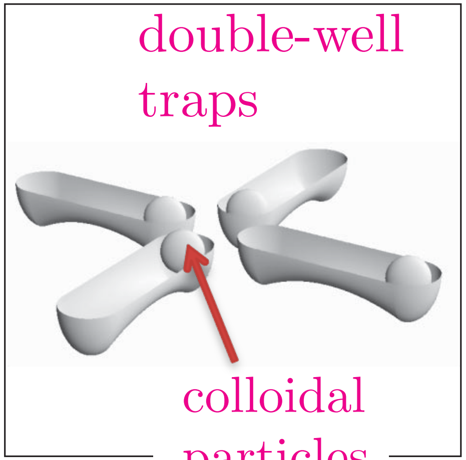
kagome ice

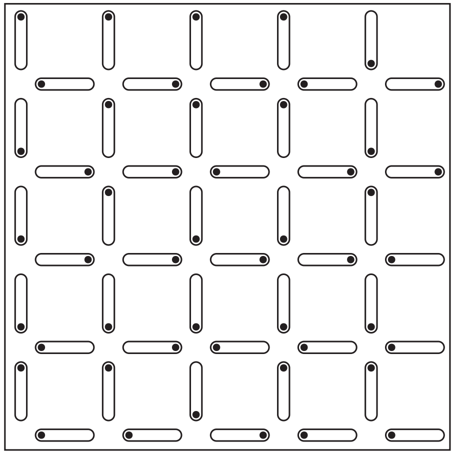


More artificial ices:

A. Libál, C. Reichhardt, & C. J. Olson Reichhardt (PRL 2006)

- charged colloidal particals on arrays of optical traps

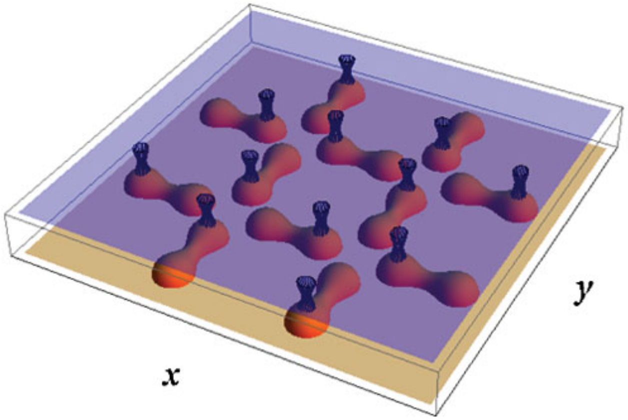


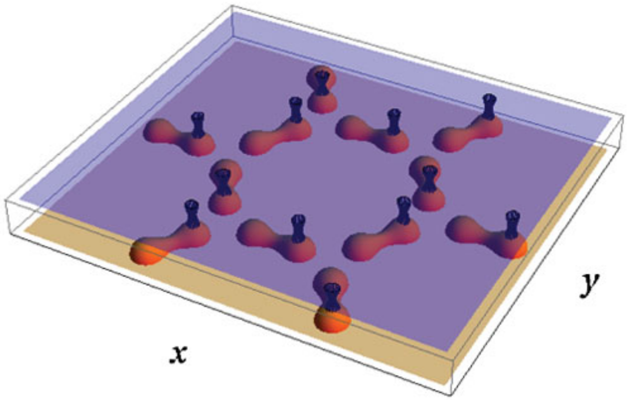


- vortices in nanostructured superconductors

A. Libál, C. J. Olson Reichhardt, & C. Reichhardt (PRL 2009)

double-well pinning sites



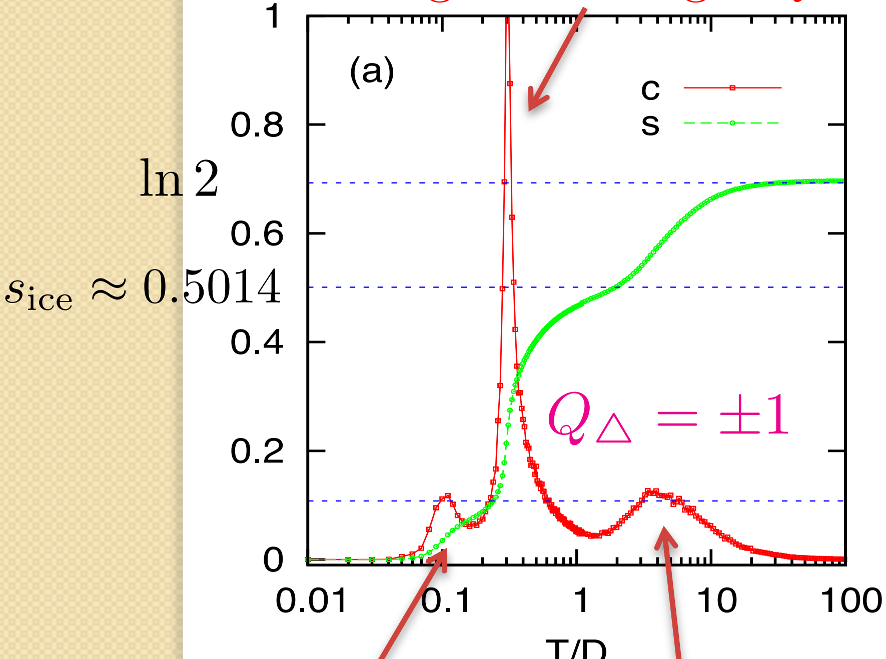


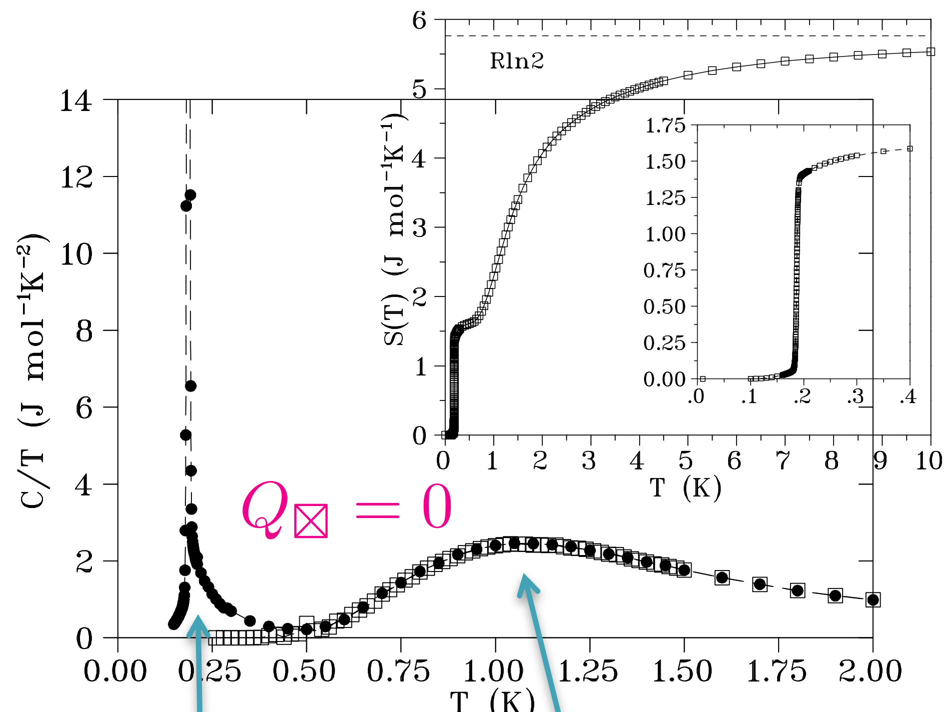
Kagome vs. Pyrochlore spin ice

- Minimizing magnetic charges Q:
  $\Rightarrow Q_{\triangle} = \pm 1$ in kagome vs. $Q_{\boxtimes} = 0$ in pyrochlore
- Two-stage ordering in kagome spin ice.

(G.-W. Chern *et al.*, PRL 2011)

ordering of
magnetic charges $Q_{\triangle}$





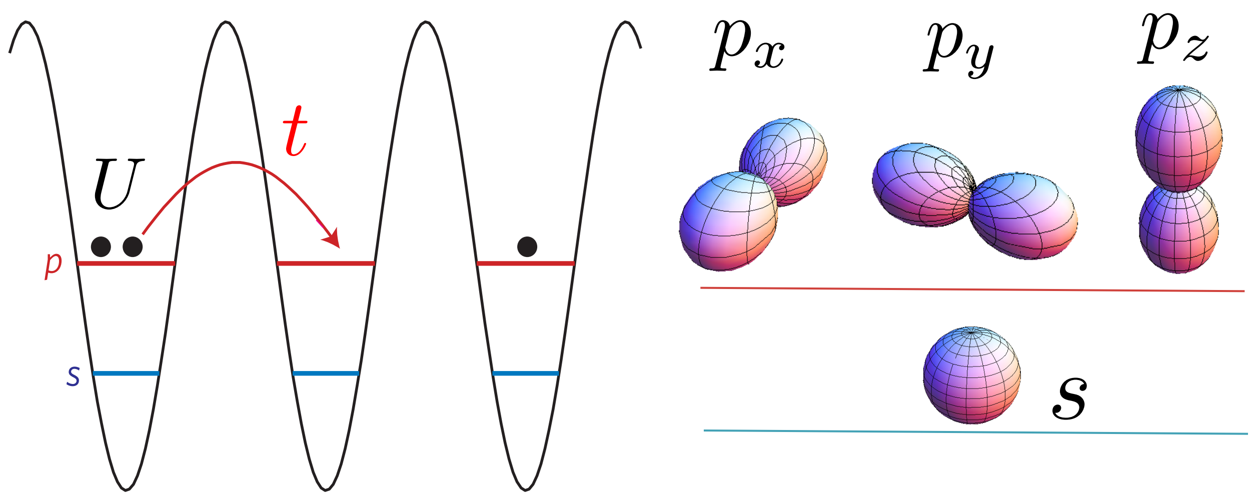
magnetic
transition

crossover
to spin ice

magnetic
transition

crossover
to spin ice

New frustrated systems on cold-atom optical lattices

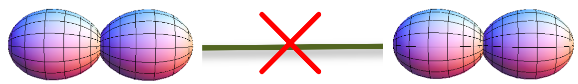


- Increasing $U/t \Rightarrow$ Mott-insulating phase of cold atoms.
- Loading *spinless* (or polarized) fermions: 2 atoms per site.

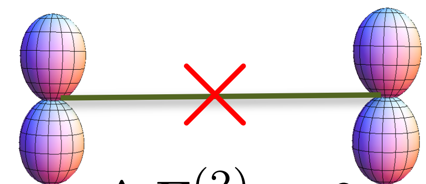
$\Rightarrow$ Remaining degrees of freedom:
localized orbitals (p_x, p_y, and p_z).

Orbital exchange physics

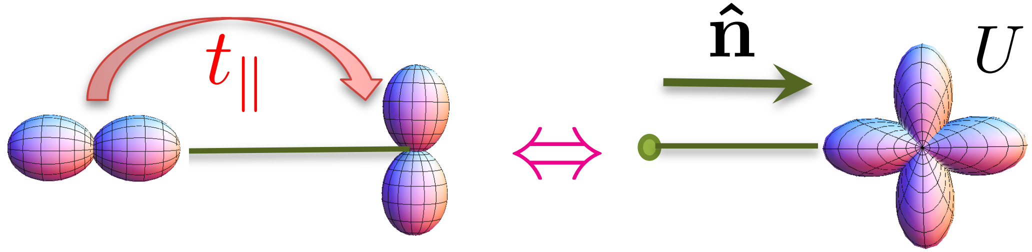
- Energy gain through 2nd-order perturbations:

(a) 

$\Delta E^{(2)} = 0.$

(b) 

$\Delta E^{(2)} = 0.$

(c) 

$\Delta E^{(2)} = -\frac{t_{\parallel}^2}{U}$

- Exchange Hamiltonian: $|\hat{\mathbf{n}}\rangle = \hat{n}_x|p_x\rangle + \hat{n}_y|p_y\rangle + \hat{n}_z|p_z\rangle$

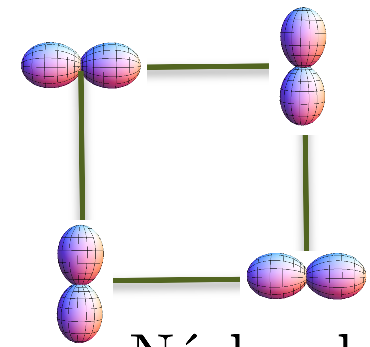
$$H_{ij} = -J\left[P_i^{\hat{\mathbf{n}}}(1 - P_j^{\hat{\mathbf{n}}}) + (1 - P_i^{\hat{\mathbf{n}}})P_j^{\hat{\mathbf{n}}}\right]$$

$P^{\hat{\mathbf{n}}} = |\hat{\mathbf{n}}\rangle\langle\hat{\mathbf{n}}|$: Projection operator of active orbital along $\hat{\mathbf{n}}$

2D Square and honeycomb lattices

- Square lattice: AF Ising model

$$H = J \sum_{\langle ij \rangle} \tau_i \tau_j \qquad \tau_i = \begin{cases} +1 & : p_x \\ -1 & : p_y \end{cases}$$

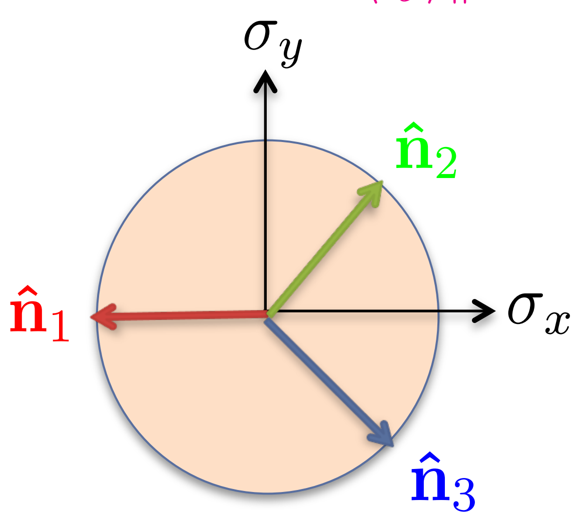


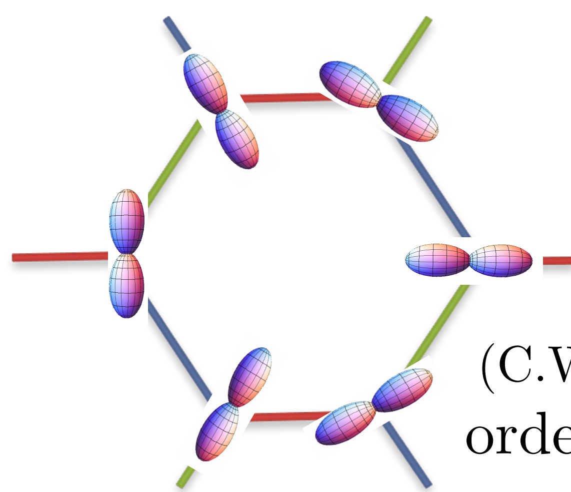
Néel order

- Honeycomb lattice: Quantum 120° model:

$$H_{120^\circ} = J \sum_{m=1}^{3} \sum_{\langle ij \rangle \| \hat{\mathbf{n}}_m} (\vec{\sigma}_i \cdot \hat{\mathbf{n}}_m)(\vec{\sigma}_j \cdot \hat{\mathbf{n}}_m)$$

frustrated !

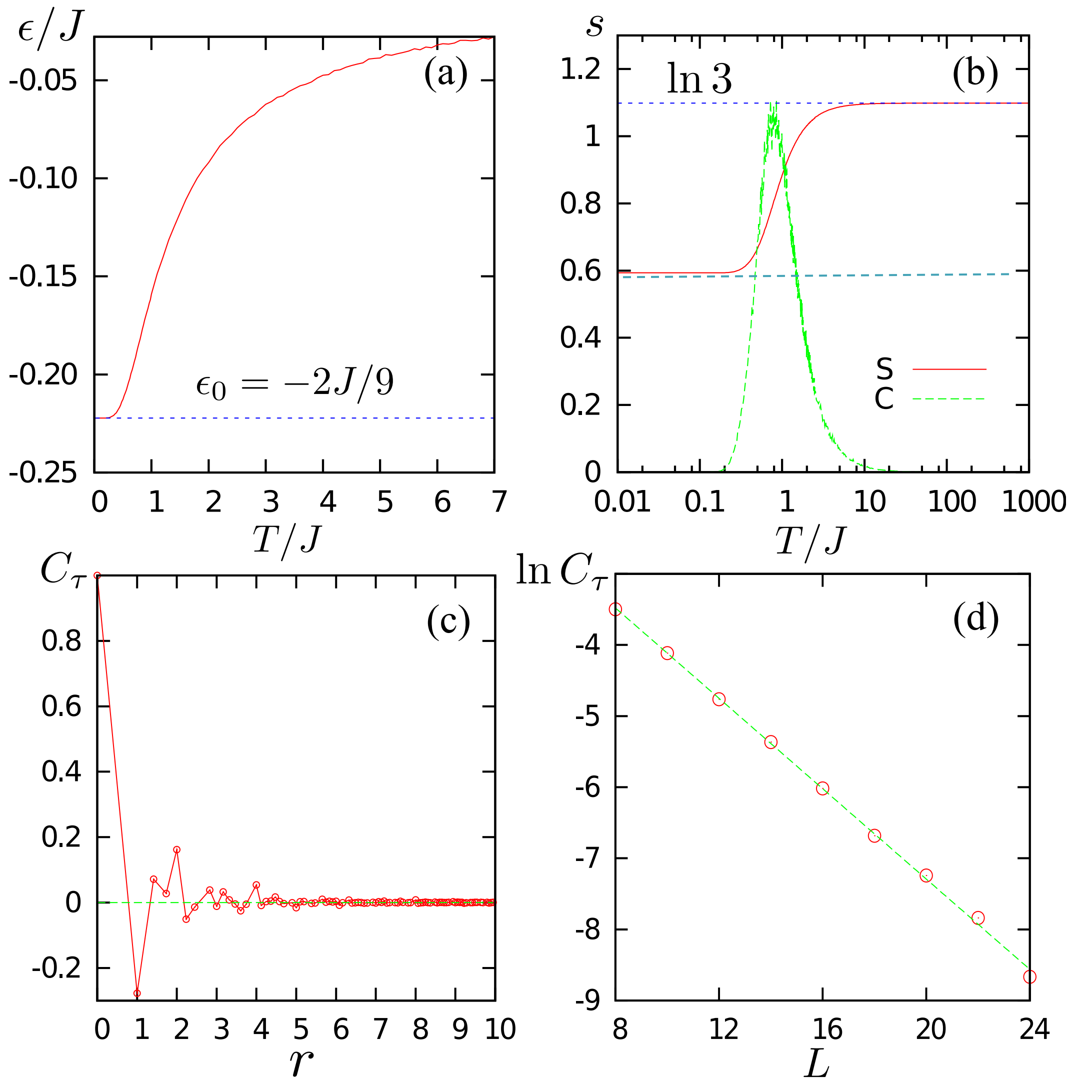


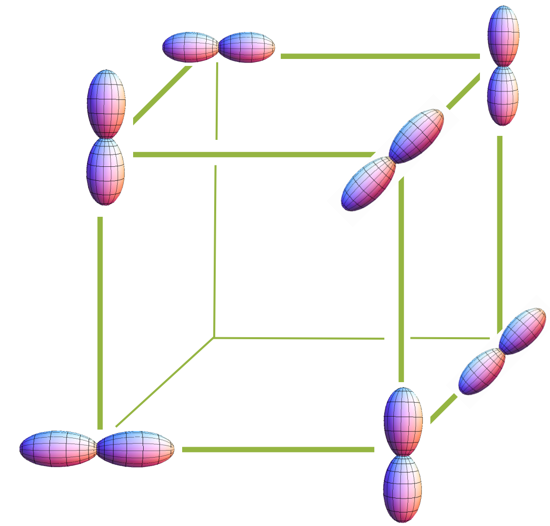


(C.Wu, PRL 2008)
order-by-disorder

3D Cubic optical lattice

Anisotropic q=3 AF Potts model:





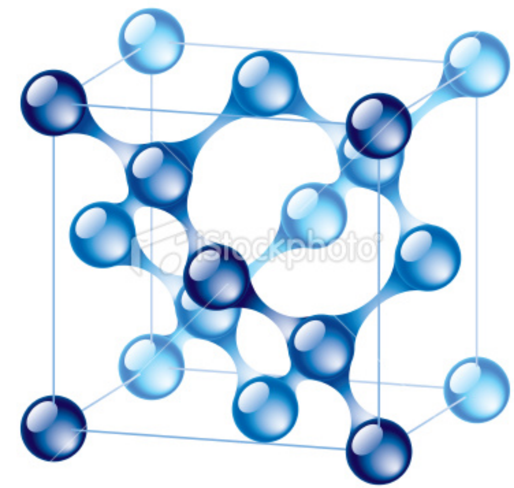
- residual entropy:

  $s_0 \approx 0.59\, k_B$

- disordered orbitals.
- exponential orbital correlation:

  $C_\tau(L) \sim \exp(-L/\xi)$

P-band Mott insulator on diamond lattice



- there are 4 distinct n.n. bonds:

$$\hat{\mathbf{n}}_0 = [111],\ \hat{\mathbf{n}}_1 = [1\bar{1}\bar{1}],$$
$$\hat{\mathbf{n}}_2 = [\bar{1}1\bar{1}],\ \hat{\mathbf{n}}_3 = [\bar{1}\bar{1}1]$$

- orbital projectors along the 4 neighbors:

$$P_m = \frac{1}{3}\left(\hat{1} + \sqrt{3}\,\vec{\mu}\cdot\hat{\mathbf{n}}_m\right)$$

- pseudovector $\vec{\mu} = (\mu_x, \mu_y, \mu_z)$

$$\mu_x = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\mu_y = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

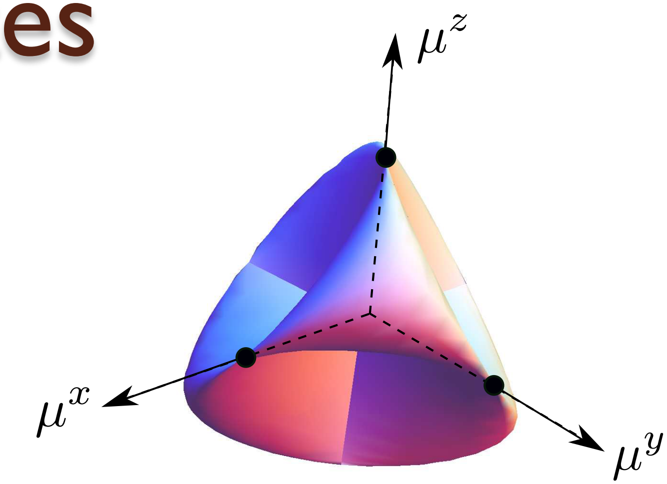
$$\mu_z = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\Rightarrow$ Quantum 'tetrahedral' Hamiltonian for pseudovectors $\{\vec{\mu}_i\}$:

$$H_{\rm ex} = J\sum_{m=0}^{4}\sum_{\langle ij\rangle\|\hat{\mathbf{n}}_m}(\vec{\mu}_i\cdot\hat{\mathbf{n}}_m)(\vec{\mu}_j\cdot\hat{\mathbf{n}}_m)$$

$([\mu_\alpha, \mu_\beta] \neq 0,\ \text{for } \alpha \neq \beta)$

Mean-field ground states



- Gutzwiller products ansatz:

$$|\Psi\rangle = \prod_{i=1}^{N} \otimes |\theta_i, \phi_i\rangle$$

- Single-site wavefunction:

$$|\theta, \phi\rangle = \sin\theta\cos\phi|p_x\rangle + \sin\theta\sin\phi|p_y\rangle + \cos\theta|p_z\rangle$$

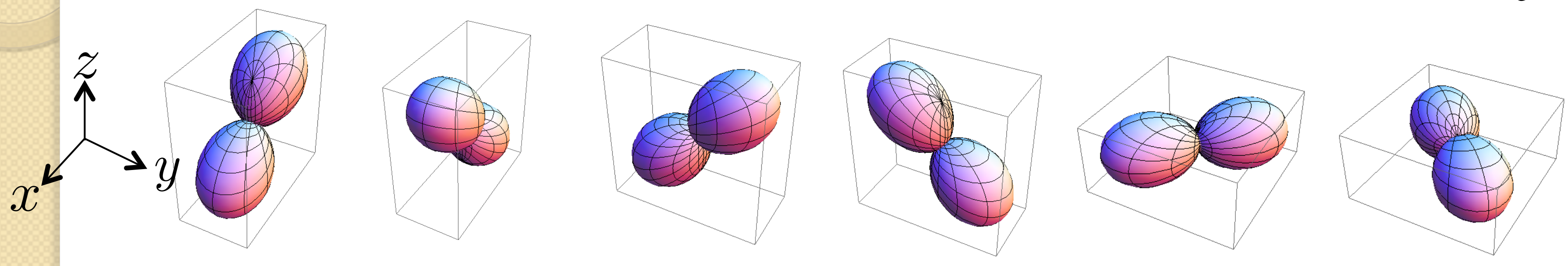
$$\Rightarrow \langle\,\vec{\mu}\,\rangle = \left(\sin 2\theta \sin\phi,\ \sin 2\theta\cos\phi,\ \sin^2\theta\sin 2\phi\right)$$

- Monte Carlo minimization of $E_{\mathrm{mf}} = \langle\Psi|H_{\mathrm{eff}}|\Psi\rangle$:
  - *Macroscopic degeneracy* of Gutzwiller ground states.
  - pseudovector $\langle\vec{\mu}_i\rangle = \pm\vec{x}$, $\pm\vec{y}$, and $\pm\vec{z}$ in the ground state.
  - Each bond $\langle ij\rangle$ has exact energy $E_{ij} = -J/3$.

Ground-state structure:

- Orbitals in the ground states:

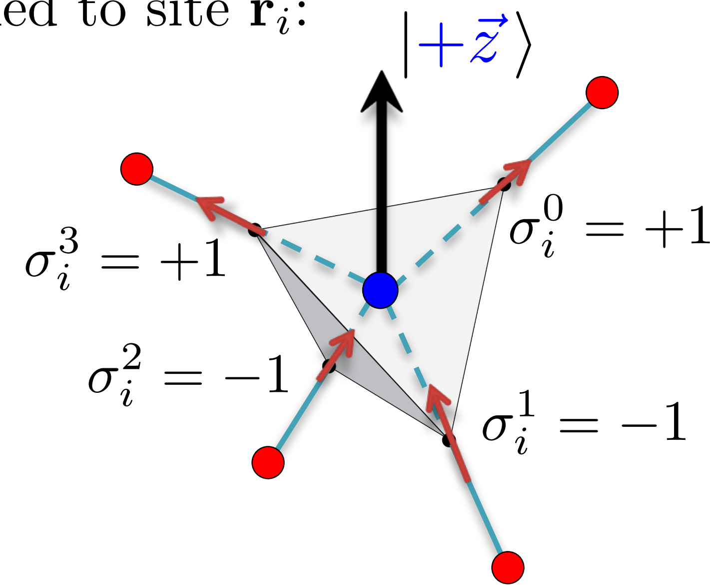
$$|\pm\vec{x}\,\rangle = |p_y\rangle \pm |p_z\rangle \quad |\pm\vec{y}\,\rangle = |p_z\rangle \pm |p_x\rangle \quad |\pm\vec{z}\,\rangle = |p_x\rangle \pm |p_y\rangle$$



- Ising variables for the 4 bonds attached to site $\mathbf{r}_i$:

$$\sigma_i^m = \sqrt{3}\langle \hat{\mathbf{n}}_m \cdot \vec{\mu}_i \,\rangle = \pm 1$$

$$(m = 0, 1, 2, 3)$$

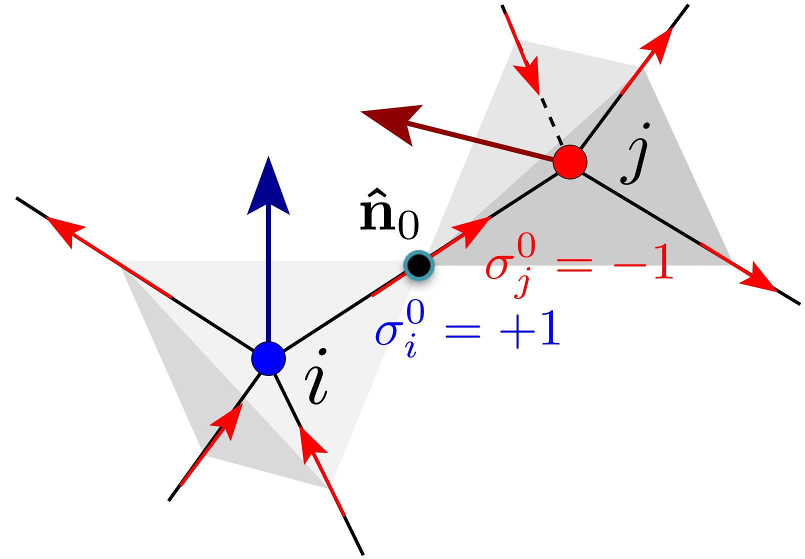


- Ground-state constraint:

$$\sigma_i^m \sigma_j^m = -1, \qquad \forall \langle ij \rangle$$

('ice' rule for orbitals !)

Exact Eigenstates



- Ground-state conditions:

  1. Cubic anisotropy:
  $\langle \vec{\mu}_i \rangle = \pm\vec{x}$, $\pm\vec{y}$, and $\pm\vec{z}$.

  2. $\{\sigma_i^m\}$ satisfy Ice rules:
  $\sigma_i^m \sigma_j^m = -1 \;\; \forall \langle ij \rangle$.

- The extensively degenerate Gutzwiller states:

$$|\Psi\rangle = |+\vec{x}\rangle_1 \otimes |-\vec{y}\rangle_2 \otimes |-\vec{x}\rangle_3 \otimes |+\vec{z}\rangle_4 \otimes |+\vec{y}\rangle_5 \otimes \cdots$$

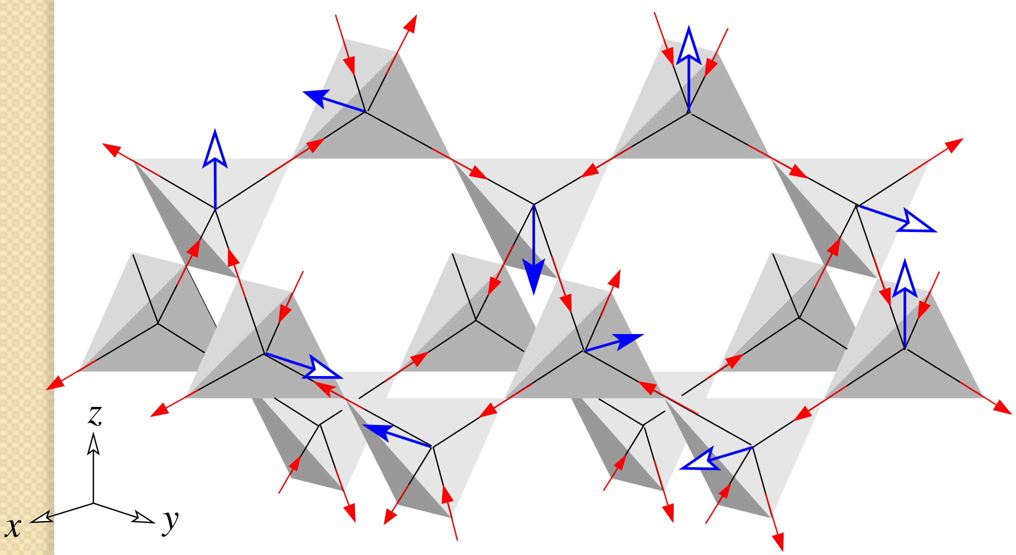
are *exact* eigenstates of the orbital exchange Hamiltonian !

$$H_{\mathrm{ex}}|\Psi\rangle = -\frac{2}{3} N J |\Psi\rangle$$

also confirmed by exact diagonalization.

Mapping to pyrochlore spin ice:

- How to characterize the degeneracy of the Gutzwiller ground states ?
- *Pyrochlore* is the "medial" lattice of the diamond structure $\Rightarrow$ placing spins at the *bond midpoints* of diamond lattice.

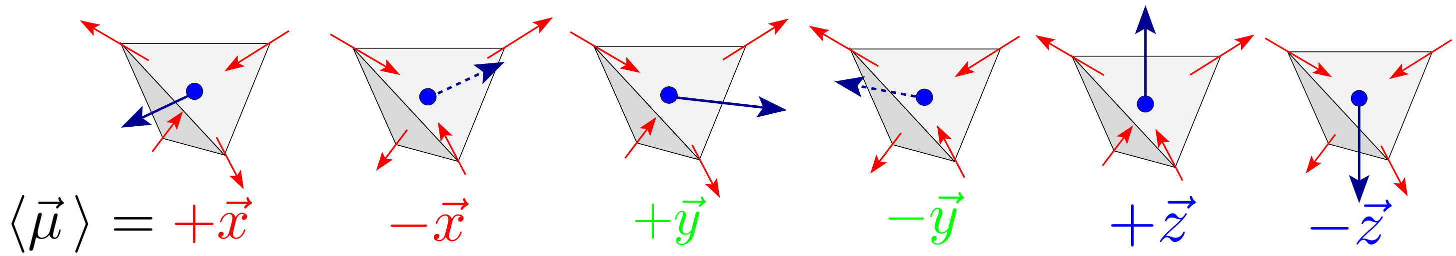


$$\mathbf{S}_{\langle ij \rangle} = +\sigma_i \hat{\mathbf{n}}_m = -\sigma_j \hat{\mathbf{n}}_m$$

- a 1-to-1 mapping of orbital ground state and spin-ice state on pyrochlore !

Orbital Coulomb phase (orbital ice)

- The six cubic directions of $\langle \vec{\mu} \rangle$ are mapped to the six 2-in-2-out ice states.



- Pauling estimate of entropy density: $s_0 = k_B \ln \frac{3}{2} \approx 0.405\, k_B$
- Dipolar-like power-law orbital correlations:

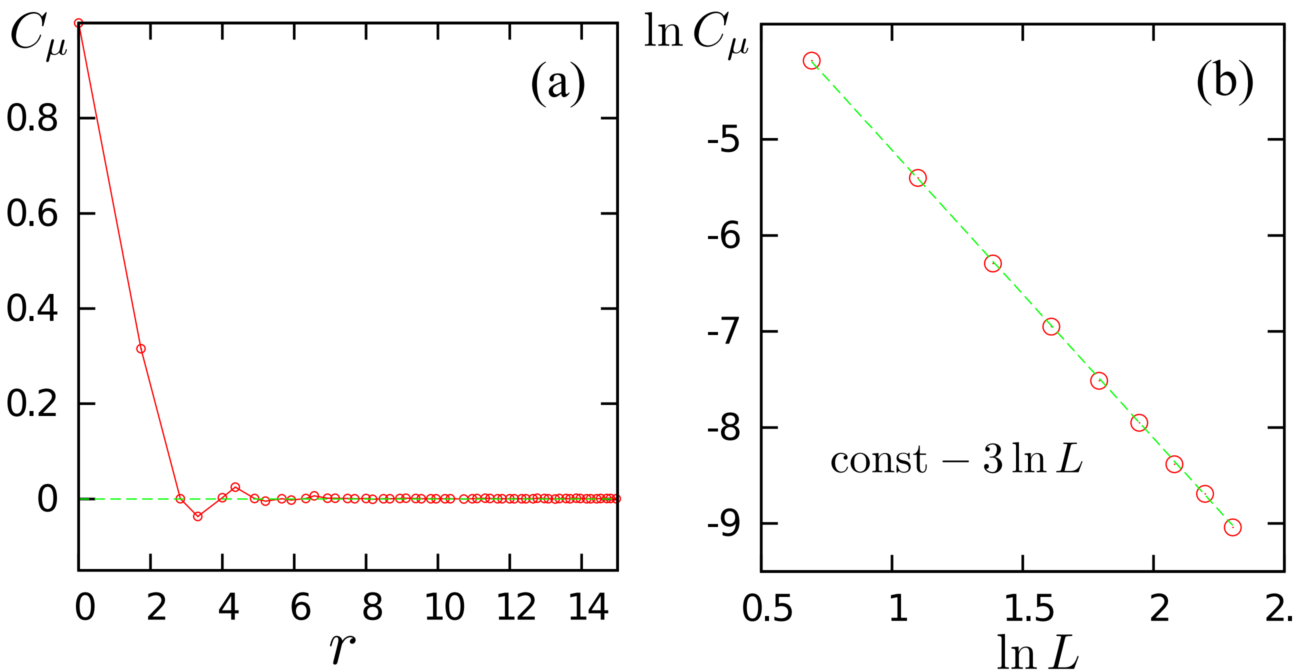
The pseudovector plays the role of the emergent 'magnetic field':

$$\mathbf{B}(\mathbf{r}_i) \approx \pm \langle \vec{\mu}_i \rangle \quad \Rightarrow \quad \text{ice rule: } \nabla \cdot \mathbf{B}(\mathbf{r}) = 0$$

$$\langle \mu_i(\mathbf{r})\, \mu_j(0) \rangle \sim \pm \frac{3x_i x_j - r^2 \delta_{ij}}{r^5} \sim \frac{1}{r^3}$$

Orbital correlation functions

- Classical Monte Carlo simulations with non-local 'loop' updates on pyrochlore spin ice
- Correlation function $C_\mu(r) = \langle \vec{\mu}(\mathbf{r}) \cdot \vec{\mu}(0) \rangle$

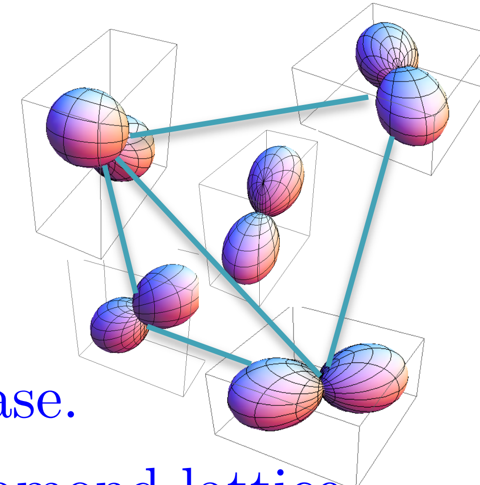


Liquid-like short-range correlation

Finite-size scaling (L: system size)

$$C_\mu(L/2) \sim \frac{1}{L^3}$$

Conclusion and Outlook



- A orbital analog of ice.
  - a first example of orbital Coulomb phase.
  - could possibly be realized in optical diamond lattice.
- *Exact* many-body ground states which form a degenerate ice-manifold of a nontrivial quantum Hamiltonian.

$$|\Psi\rangle = |+\vec{x}\rangle_1 \otimes |-\vec{y}\rangle_2 \otimes |-\vec{x}\rangle_3 \otimes |+\vec{z}\rangle_4 \otimes |+\vec{y}\rangle_5 \otimes \cdots$$

$\Rightarrow$ Experimental signature of orbital ice: structure factor, time-of-flight measurement ...

$\Rightarrow$ (exact) Elementary excitations ? Monopole-like quasiparticles ?