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Effective interaction for correlated electrons with strong electron-phonon coupling

Johannes Bauer

Max-Planck Institute for Solid State Research, Stuttgart

Collaborations:

G Sangiovanni (TU Vienna)

O Gunnarsson (MPI Stuttgart)

AC Hewson (Imperial College London)

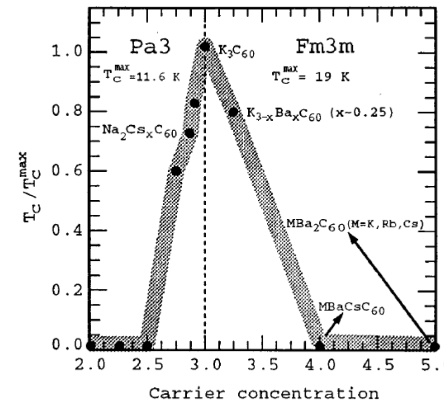
J Han (SUNY, Buffalo)

Electron-electron and electron-phonon interactions in condensed matter systems

1. Strongly coupled electron-phonon systems

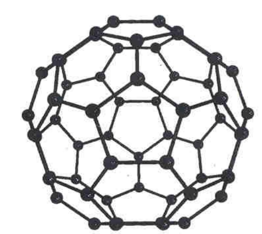
Systems, where phonon scale and electron phonon coupling is not small

- Fullerides (doped lattices of fullerenes)



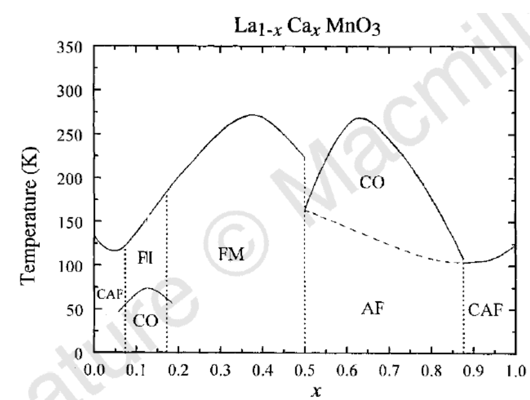
Superconducting at high critical temperatures T_c of 30 - 40°K.

O. Gunnarson, RMP **69**, 575 (1997)

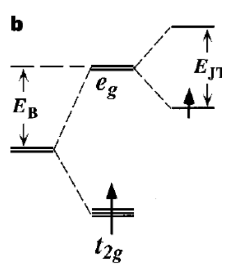


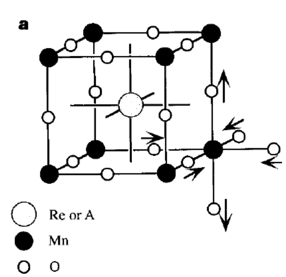
- Manganites $Re_{1-x}A_xMnO_3$

Materials with large magnetoresistivity



A.J. Millis, Nature **392**, 147 (2001)



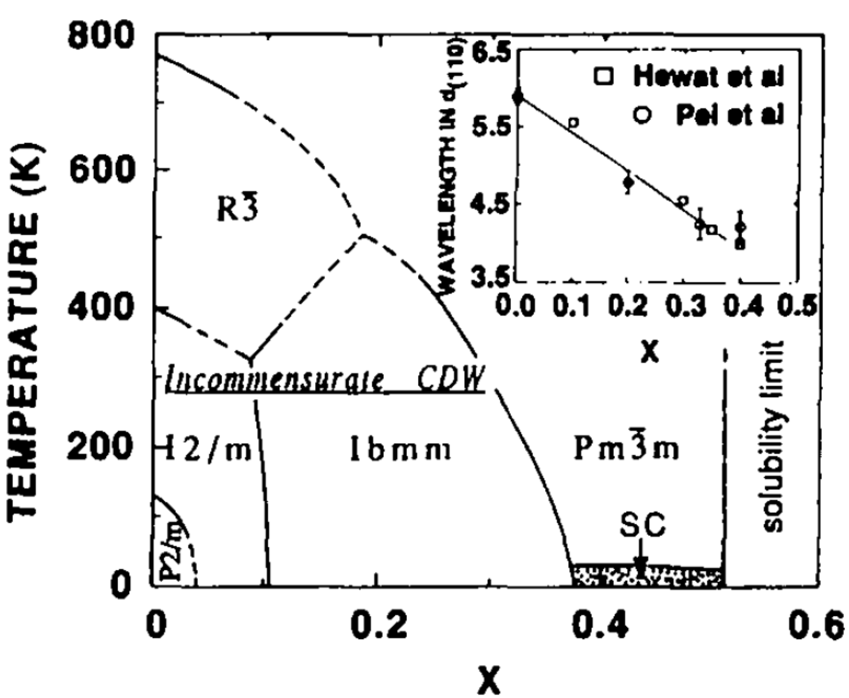


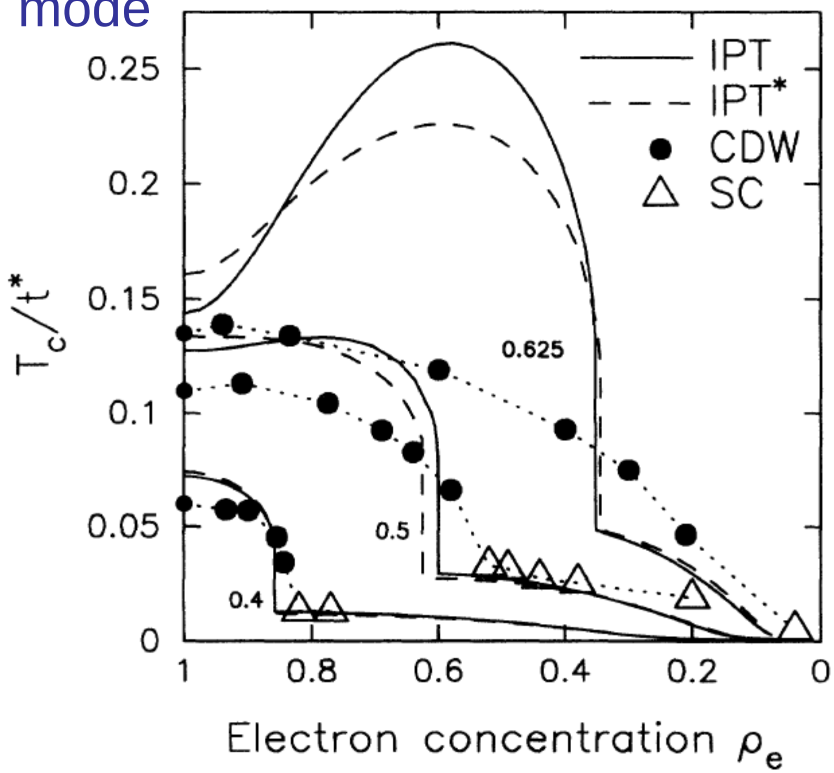
Electron-electron and electron-phonon interactions in condensed matter systems

2. Strongly coupled electron-phonon systems

- Compound $Ba_{1-x}K_xBiO_3$

Strong coupling to oxygen breathing mode





D.G. Hinks et al., Physica C **162**, 1405 (1989)

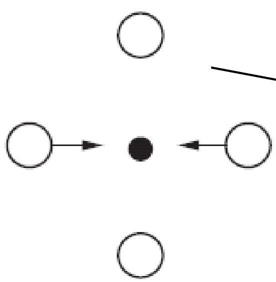
Freericks, Jarrel, PRB **50**, 6939 (1994)

Electron-electron and electron-phonon interactions in condensed matter systems

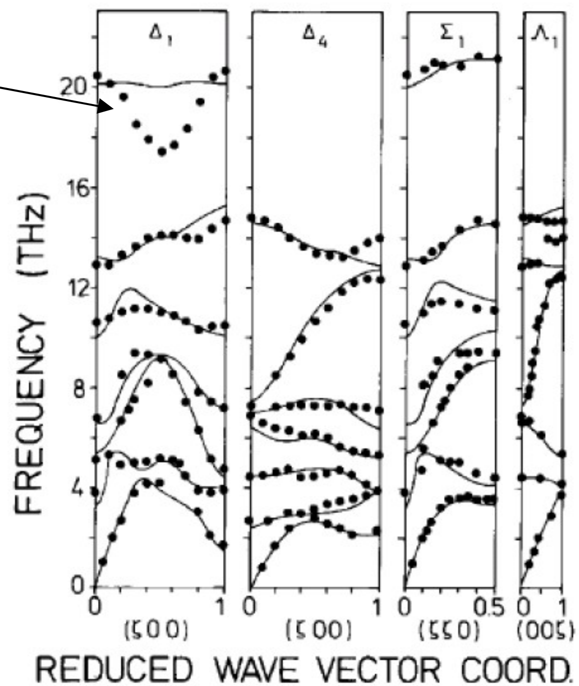
3. Strongly coupled electron-phonon systems

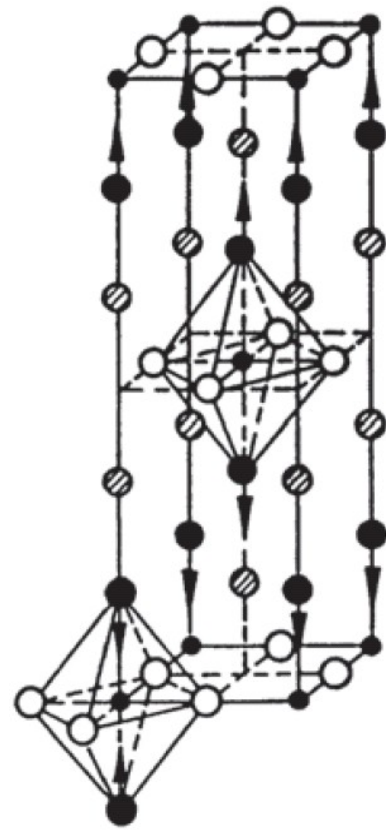
- Cuprates

Strong coupling to oxygen half breathing mode



$\mathbf{q} = (0.5,0,0)$



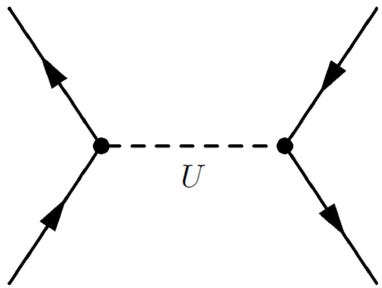


O Gunnarsson, O Rösch, J. Phys.: Cond Mat. **20**, 043201 (2008)

Interactions on different energy scales

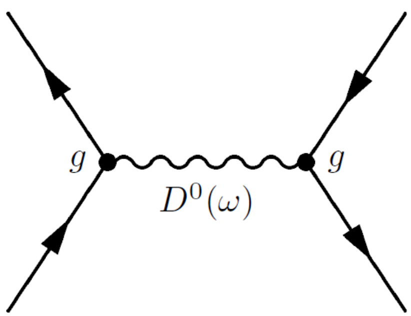
Bandwidth scale

Electronic interaction on all energy scales

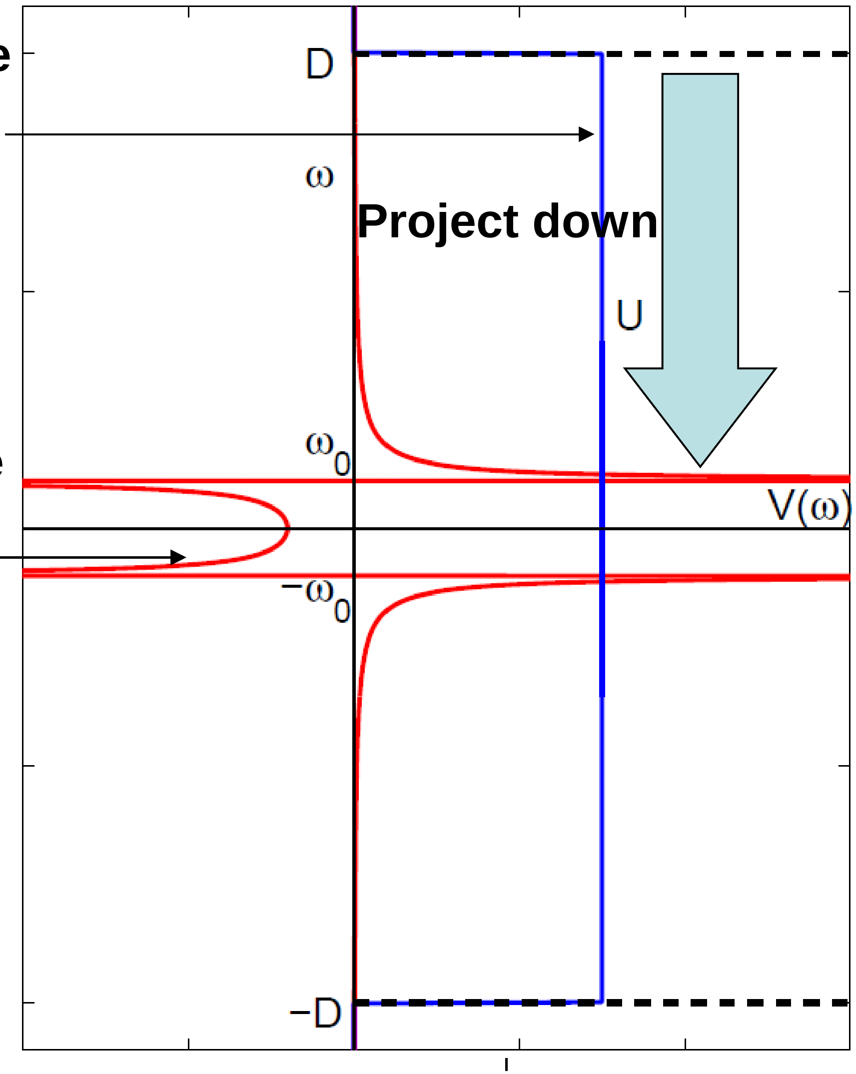


Phonon scale

Electron phonon interaction on a lower energy scale



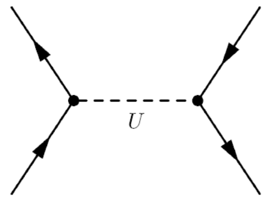
Effective interaction at low energy?



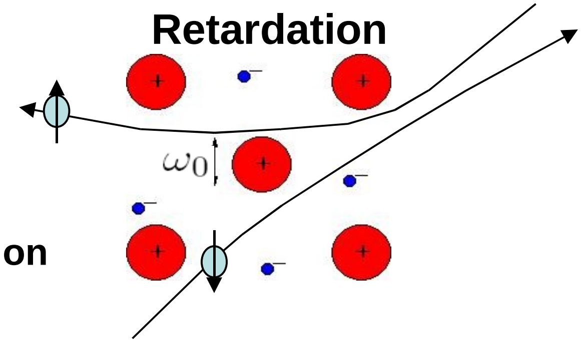
Theory of conventional superconductivity

Coulomb repulsion

μ_c Fermi surface average of screened Coulomb repulsion

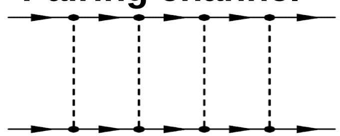


Retardation



Effective interaction on phonon scale

Pairing channel



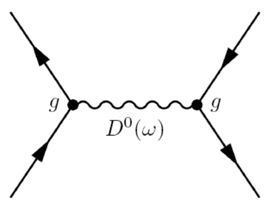
RG

$$\frac{dU(\Lambda)}{d\Lambda} = \frac{\rho_0}{\Lambda} U(\Lambda)^2$$

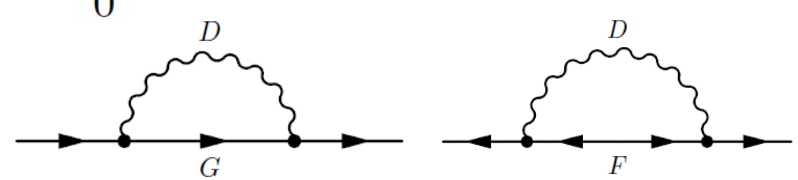
$$\boxed{\mu_c^* = \frac{\mu_c}{1+\mu_c \log(\frac{D}{\omega_0})}}$$

Attraction mediated by phonons

λ Dimensionless effective electron-electron attraction



$$\lambda = \int_0^\infty \mathrm{d}\omega \, \frac{2\alpha^2 F(\omega)}{\omega}$$



$$\boxed{T_c = \frac{\langle \omega \rangle}{1.2} \mathrm{e}^{-\frac{1.04(1+\lambda)}{\lambda - \mu_c^*(1+0.62\lambda)}}}$$

Allen, Dynes, Phys. Rev. B **12**, 905 (1975)

McMillan, Phys. Rev. **167**, 331 (1968)

Overview

1. Phase diagram: competing interactions for the Hubbard Holstein model at half filling
   - Normal state
   - With symmetry breaking
2. Kinks in the nodal dispersion of the copper-oxide superconductors
   - Recent experiments
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Hubbard-Holstein (HH) model

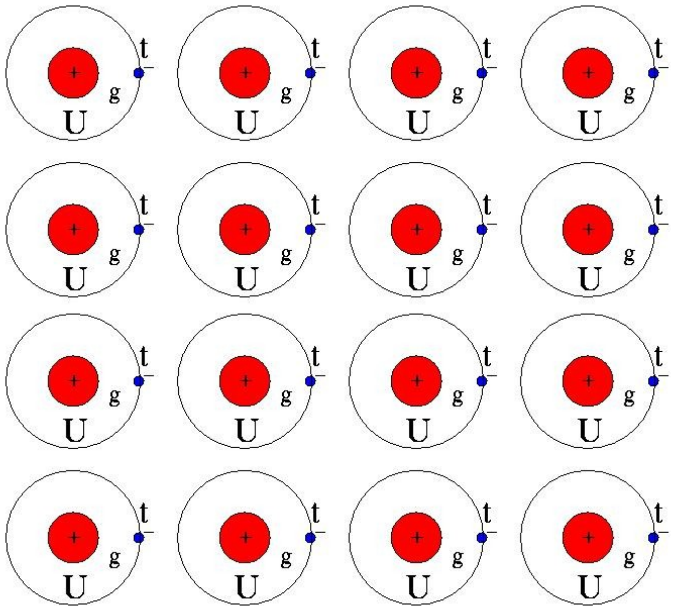
$$H = -t \sum_{i,j,\sigma} (c^\dagger_{i,\sigma} c_{j,\sigma} + \mathrm{h.c.}) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$
$$+\omega_0 \sum_i b^\dagger_i b_i + g \sum_i (b_i + b^\dagger_i) \Big(\sum_\sigma \hat{n}_{i,\sigma} - 1 \Big)$$

Describe coupling to optical phonon in the presence of strong electron-electron repulsion

U vs $\lambda = 2g^2/\omega_0$

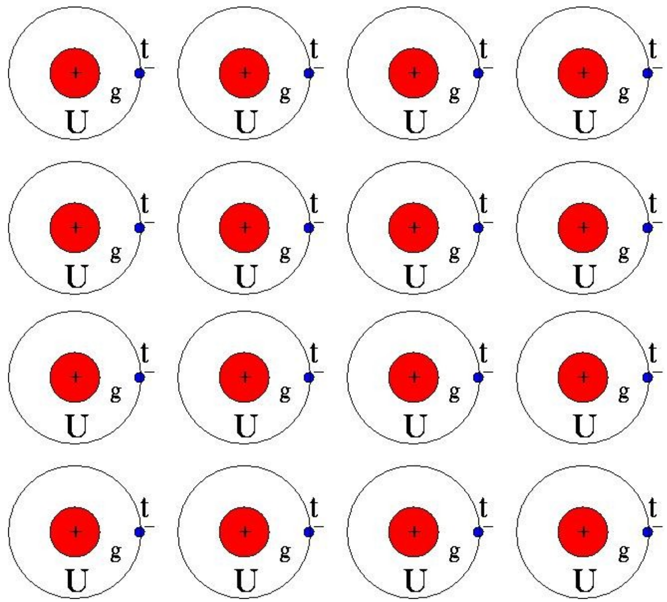
Electron scale $W = 2D = 4t \quad \rho_0 = \frac{2}{\pi D}$

Phonon scale ω_0 **renormalized** ω_0^r



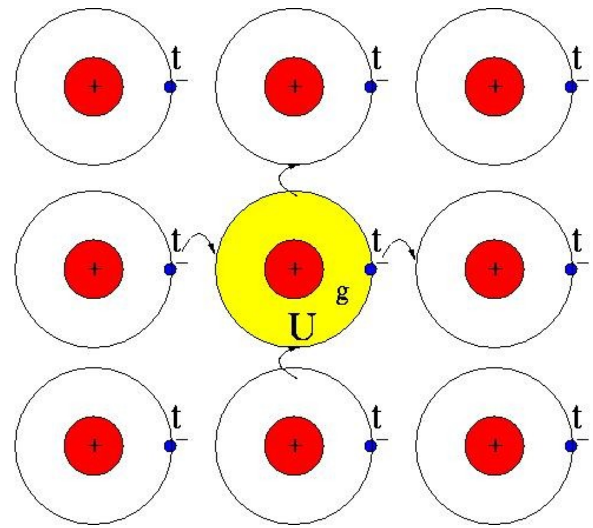
Dynamical mean field theory (DMFT) and Numerical Renormalisation group (NRG)

Hubbard-Holstein model

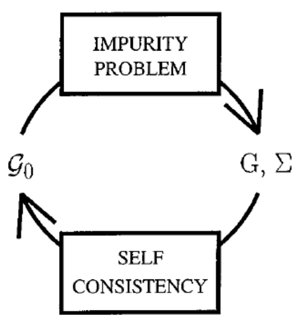


DMFT

Effective Anderson-Holstein model



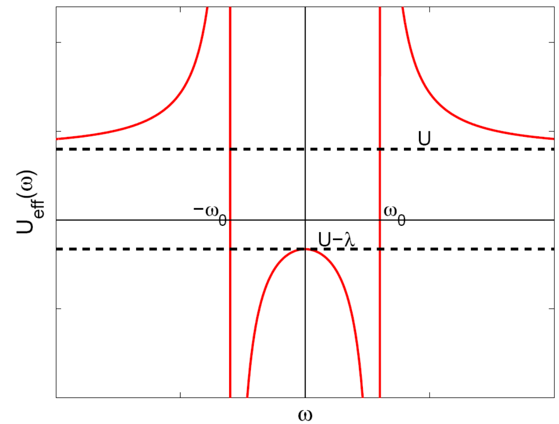
Solve with NRG



Obtain static and dynamic response functions

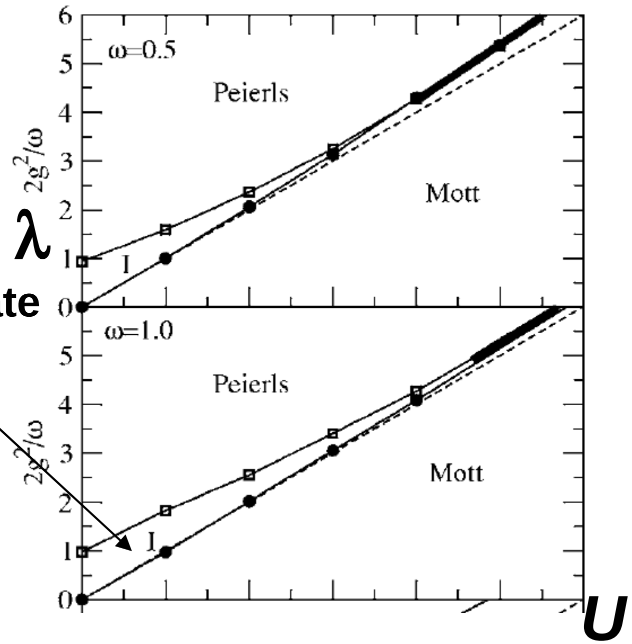
Competing interactions and phase diagram at half filling

$$\boxed{U_{\mathrm{eff}}(\omega) = U + \frac{2g^2\omega_0}{\omega^2-\omega_0^2}}$$



$$\lambda = 2g^2/\omega_0$$

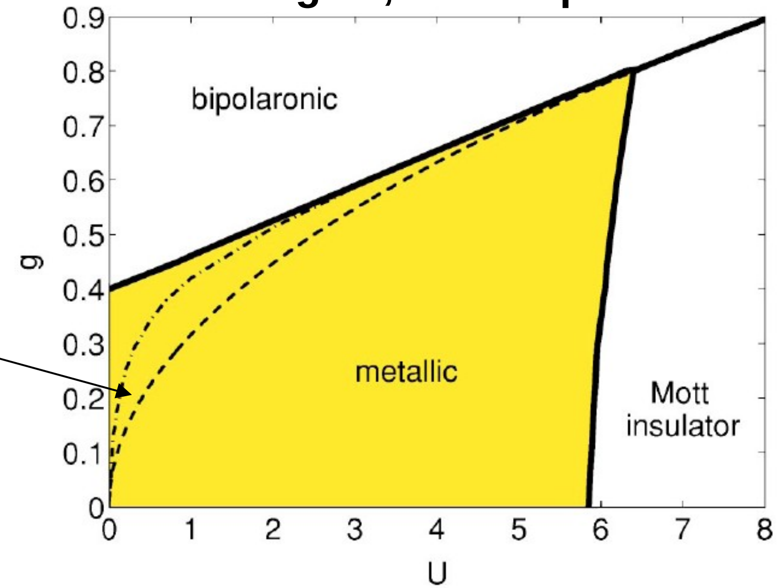
d = 1



Intermediate phase

$$U = \lambda$$

large d, normal phase



R.P. Hardikar, J.P. Clay, PRB **75**, 245103 (2007)

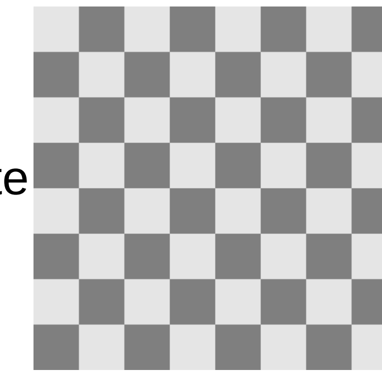
W. Koller et al., EPL **66**, 559 (2004); PRB **70**, 155103 (2004)

Large d including long range order

Commensurate order on a bipartite lattice

$$
\begin{aligned}
H_\mu &= \sum_{\boldsymbol{k},\sigma} (\varepsilon_k c^\dagger_{A,\boldsymbol{k},\sigma} c_{B,\boldsymbol{k},\sigma} + \text{h.c.}) - \sum_{\boldsymbol{k},\sigma} (\mu_\sigma c^\dagger_{A,\boldsymbol{k},\sigma} c_{A,\boldsymbol{k},\sigma} + \mu_{-\sigma} c^\dagger_{B,\boldsymbol{k},\sigma} c_{B,\boldsymbol{k},\sigma}) \\
&\quad + U \sum_i (n_{A,i,\uparrow} n_{A,i,\downarrow} + n_{B,i,\uparrow} n_{B,i,\downarrow}),
\end{aligned}
$$

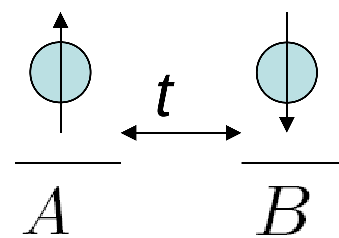
A-*B* bipartite lattice



We will consider half filling.

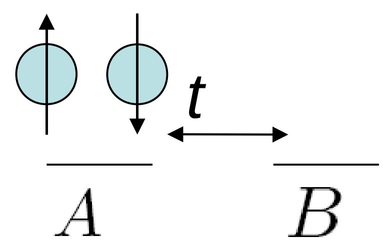
Antiferromagnetic order:

$$\boxed{\Phi_{\text{afm}} = (n_{A,\uparrow} - n_{A,\downarrow})/2}$$



Charge order:

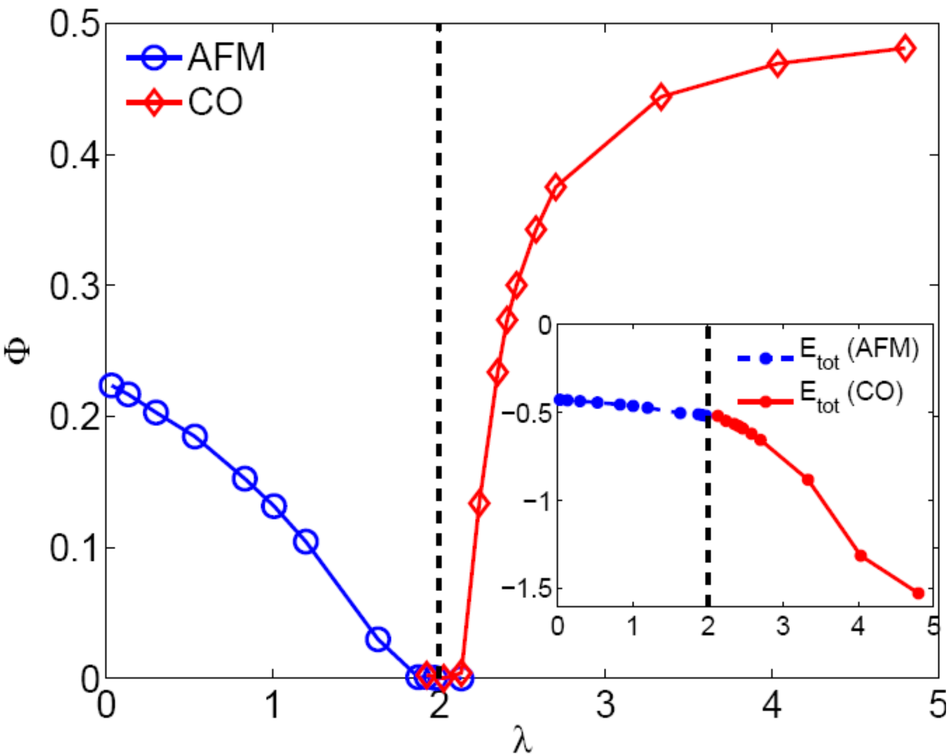
$$\boxed{\Phi_{\text{co}} = (n_A - 1)/2}$$



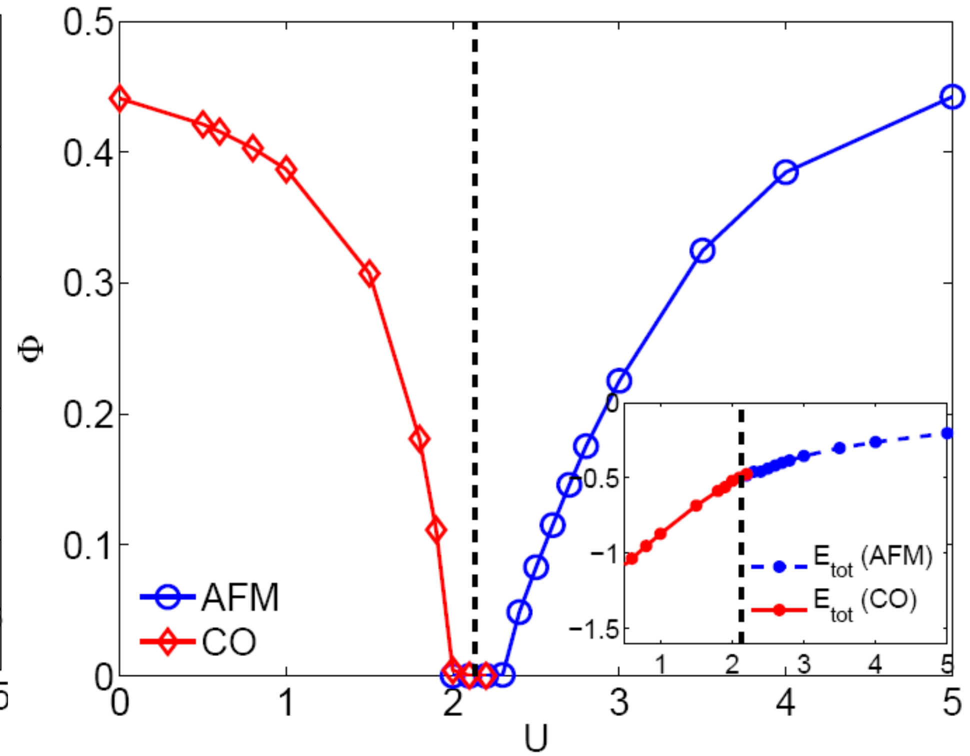
Order parameters weak coupling

Bandwidth *W* = 4, half filling $\omega_0 = 0.6$

Fixed *U* $U = 2$



Fixed λ $\lambda \simeq 2.13$

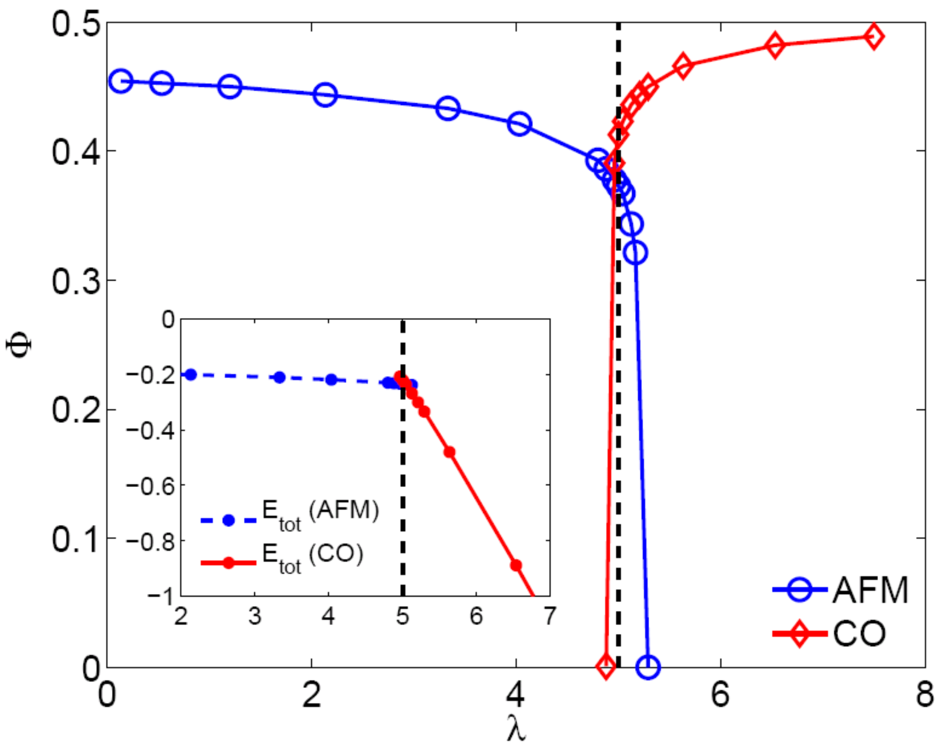


Continuous transition

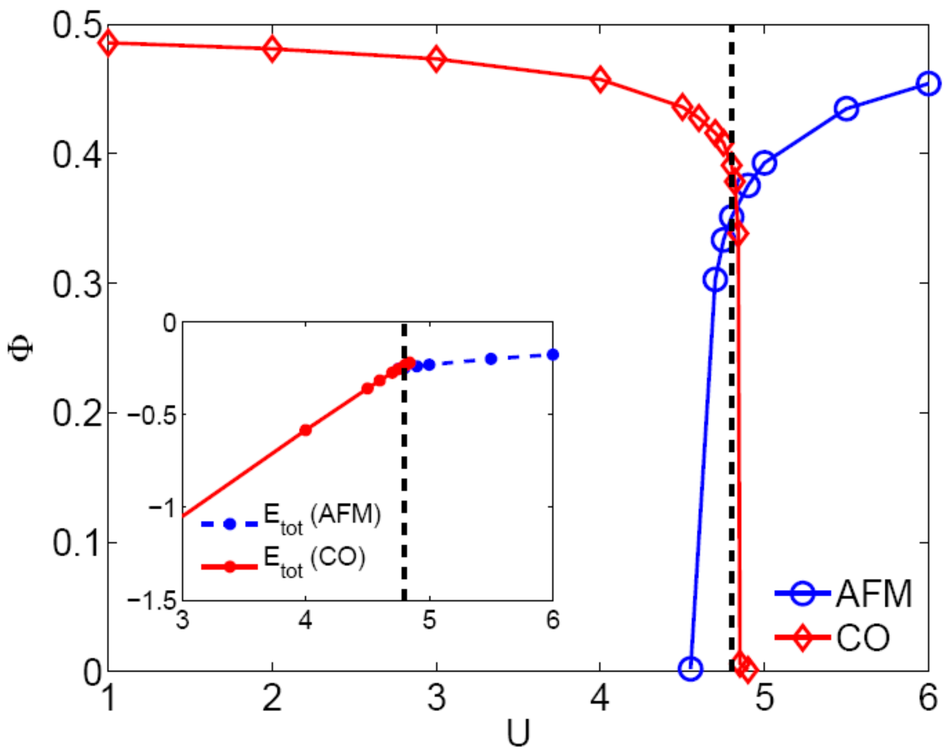
Order parameters strong coupling

Bandwidth *W* = 4, half filling $\omega_0 = 0.6$

Fixed *U* $U = 5$

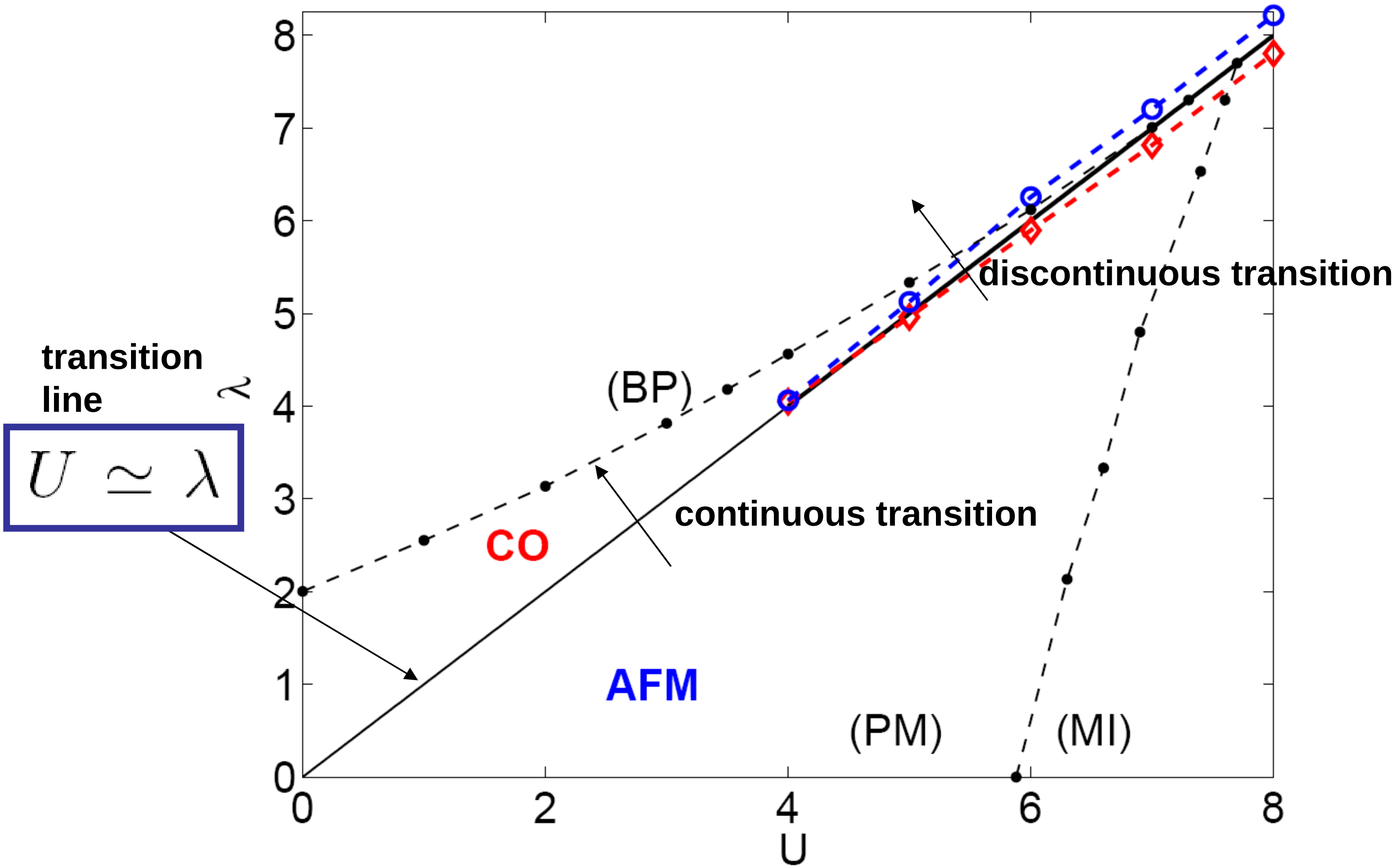


Fixed λ $\lambda \simeq 4.8$



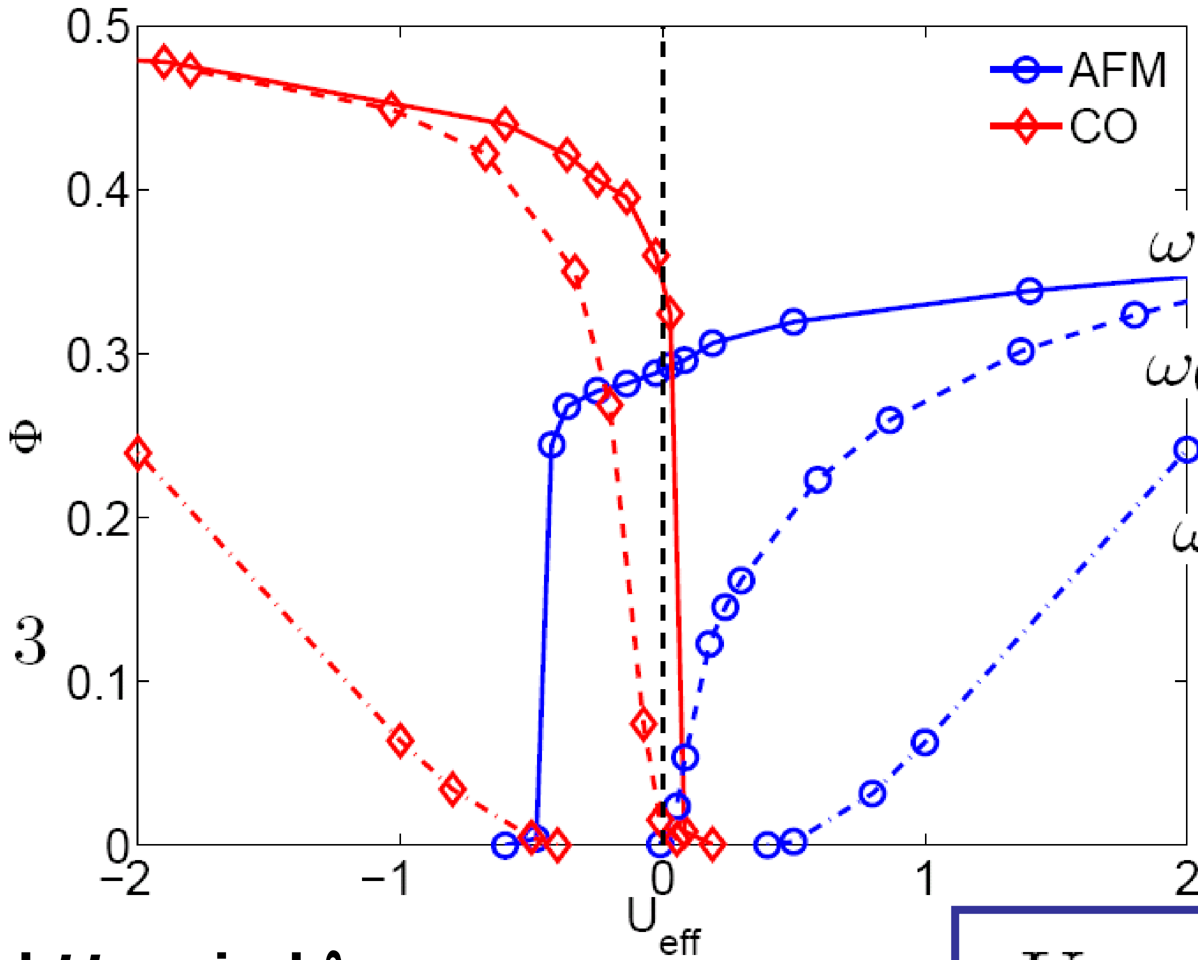
Discontinuous transition

Phase diagram of the HH model



Bandwith *W* = 4, half filling $\omega_0 = 0.6$

Role of phonon frequency



$\omega_0 = 0.2$

$\omega_0 = 0.6$

$\omega_0 \to \infty$

$U = 3$

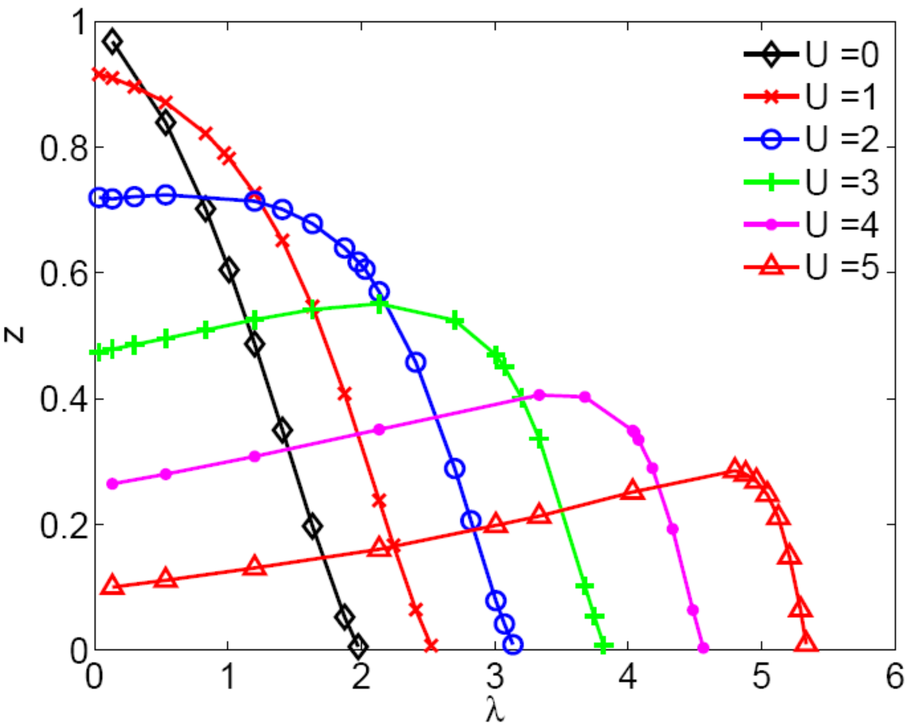
Fixed *U* varied λ

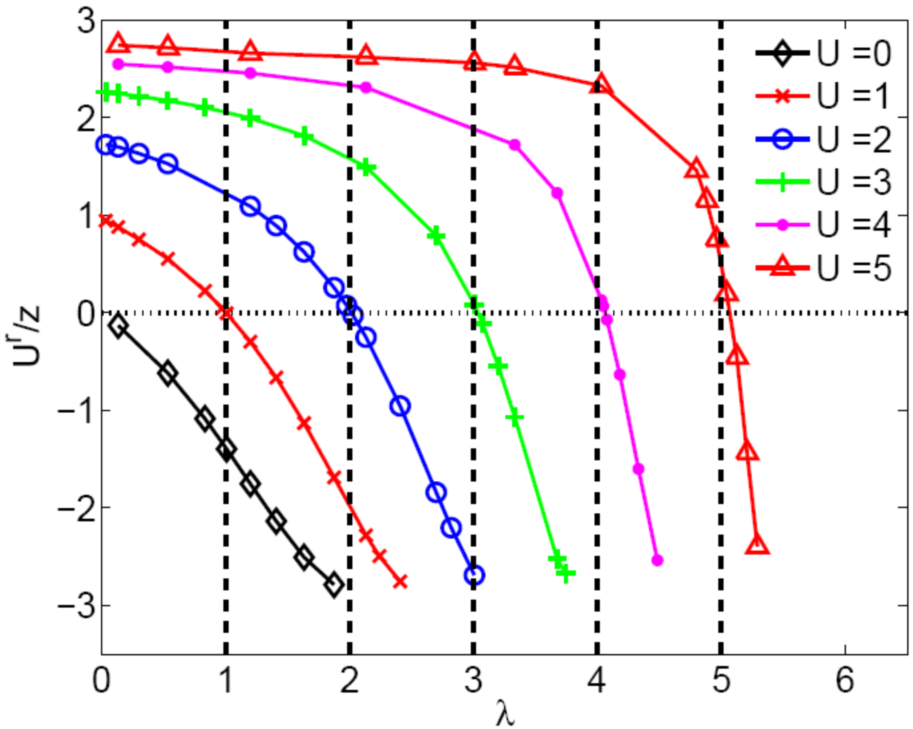
$$\boxed{U_{\text{eff}} = U - \lambda}$$

Effective mass and quasiparticle interaction in the normal state

$$z^{-1} = m^*$$

$$U^r = \tilde{U}$$





Bandwith W = 4, half filling

$\omega_0 = 0.6$

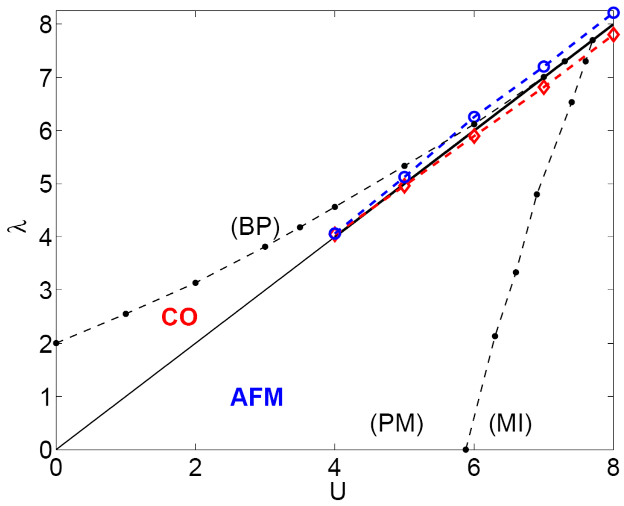
From two-particle excitations find effective quasiparticle interaction

$$\boxed{\tilde{U} \sim E_{pp} - 2E_p}$$

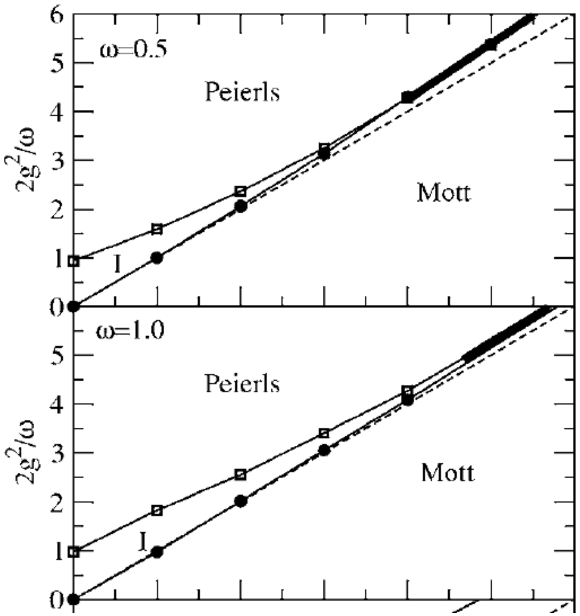
Take away message for phase diagram at half filling

- Phase boundaries are largely determined from effective interaction in terms of **bare** parameters

$$U_{\text{eff}} = U - \lambda \qquad \lambda = 2g^2/\omega_0$$



- For other quantities this is different
- It appears that interactions are renormalized in a similar way
- Phase diagram has similarity with low dimensional case



JB, AC Hewson, Phys. Rev. B **81**, 235113 (2010)

JB, EPL **90**, 27001(2010)

Effective one band HH model for materials

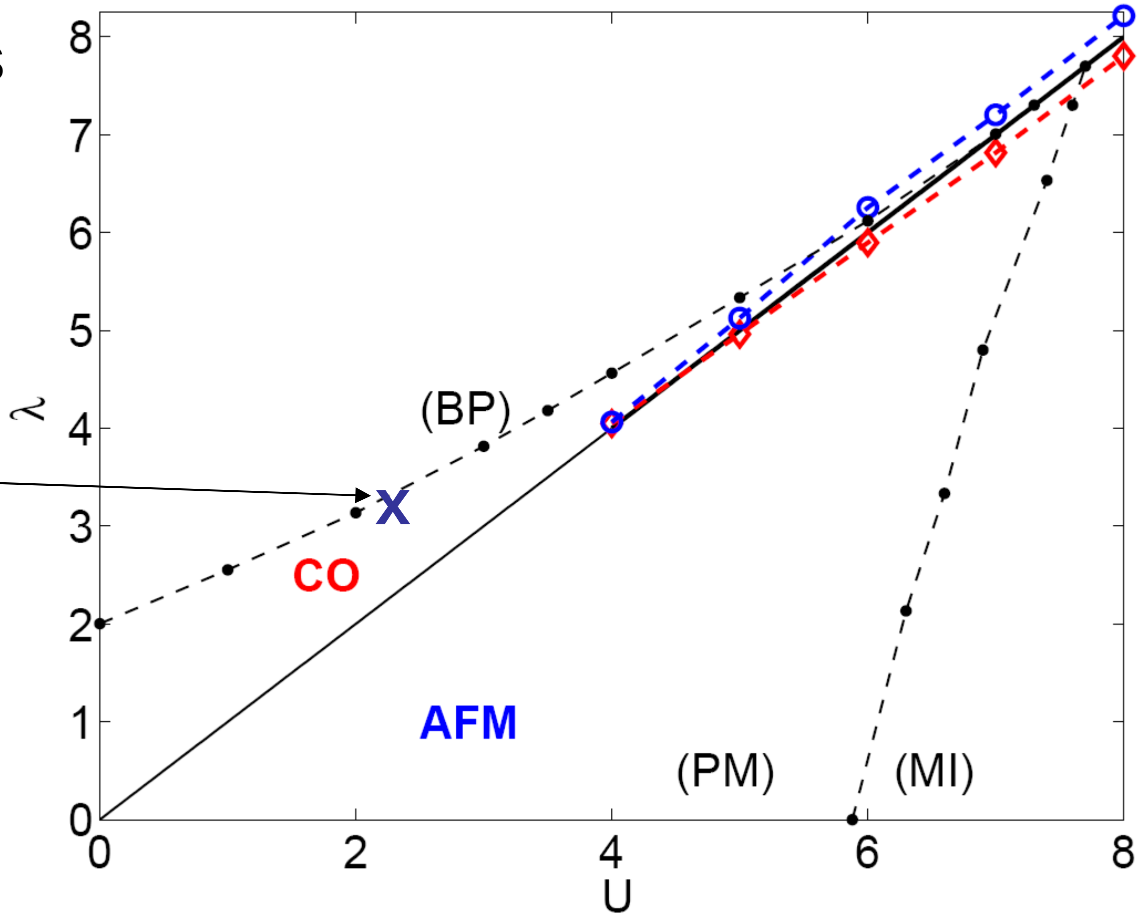
- Compound $Ba_{1-x}K_xBiO_3$

LDA estimates

$$\lambda\rho_0(0) \simeq 1$$

$$\lambda \simeq 3.1$$

$$U \approx 2.2$$



G. Vielsack, W. Weber, PRB **54**, 6614 (1996)

Liechtenstein et al. , PRB **44**, 5388 (1991)

JB, EPL **90**, 27001(2010)

Overview

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Kinks in nodal dispersion of cuprates

Kinks: abrupt changes in the electronic dispersion relation

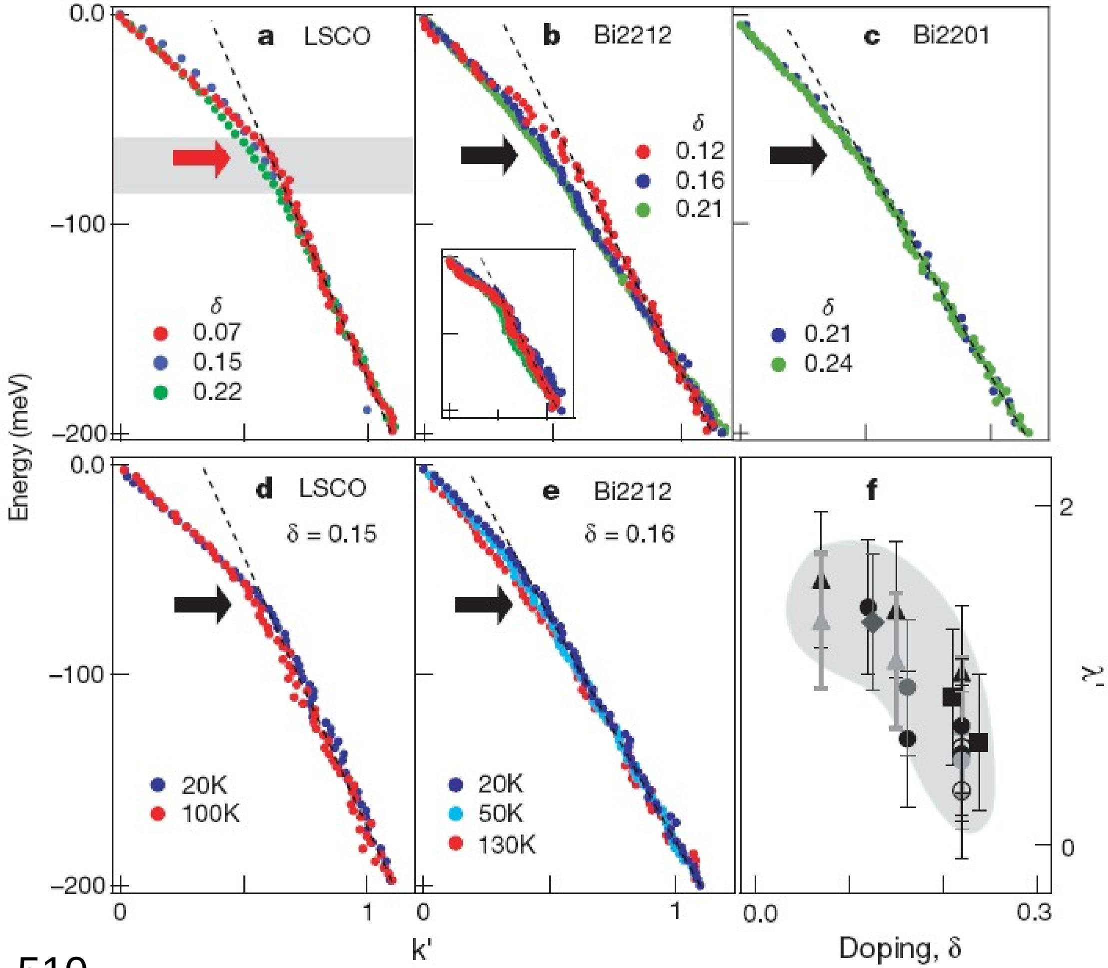
Kink energy

$$\omega_0 = 50 - 80\mathrm{meV}$$

Kink magnitude

$$m_k = 1 + \lambda' \approx 2$$

ARPES data



Lanzara et al., Nature **412**, 510 (2001)

Different possible origins of kinks in the dispersion

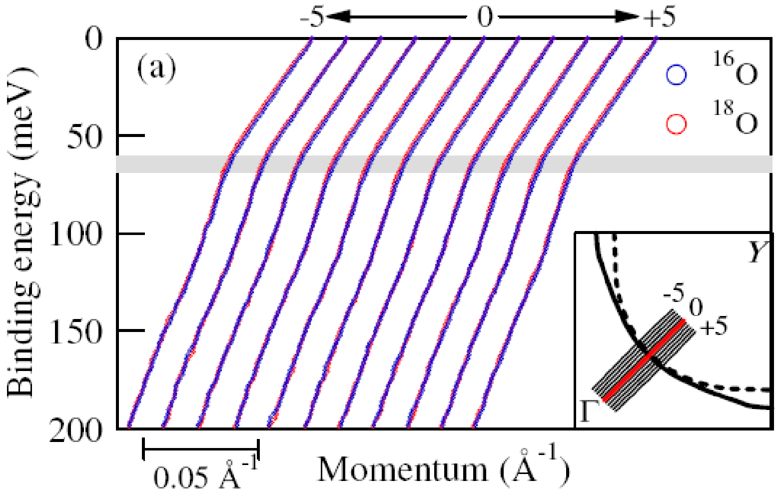
- Coupling to a phonon mode
  - Lanzara et al., Nature (2001)
  - Cuk et al., PSS (2005)
  - Graf et al., PRL (2008)

- Coupling to spin fluctuations
  - Johnson et al., PRL (2001)
  - Kaminski et al., PRL (2001)
  - Manske et al., PRL (2001)
  - Chubukov et al., PRB (2004)
  - Eschrig, Adv. Phys. (2006)

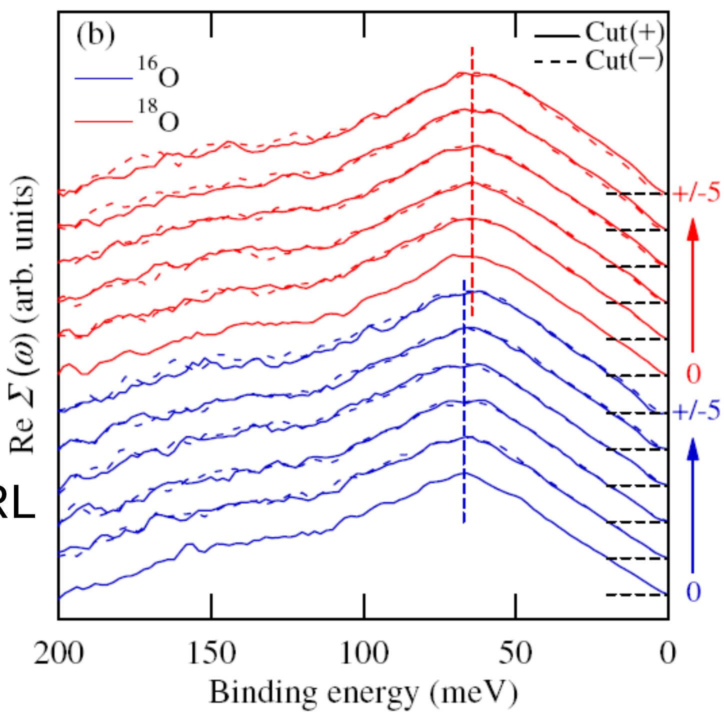
- Purely caused by electronic correlations
  - Byzcuk et al., Nature Physics (2007)

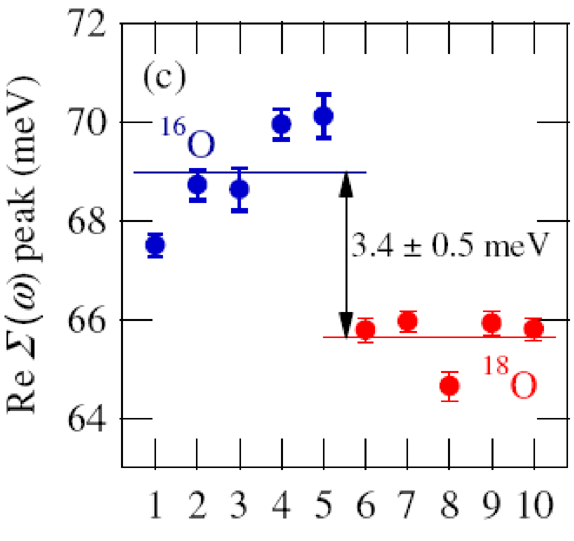
Case in favour of phonons: Isotope effect for kinks

ARPES data



Extract electronic self-energy

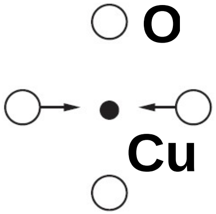




H. Iwasawa et al, PRL **101**, 157005 (2008)

Isotope substitution leads to softening of phonon

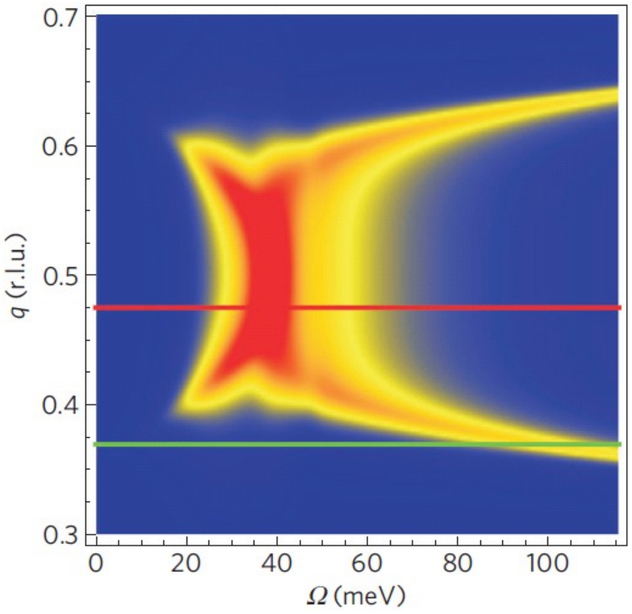
$$\Omega(1-\sqrt{16/18})$$



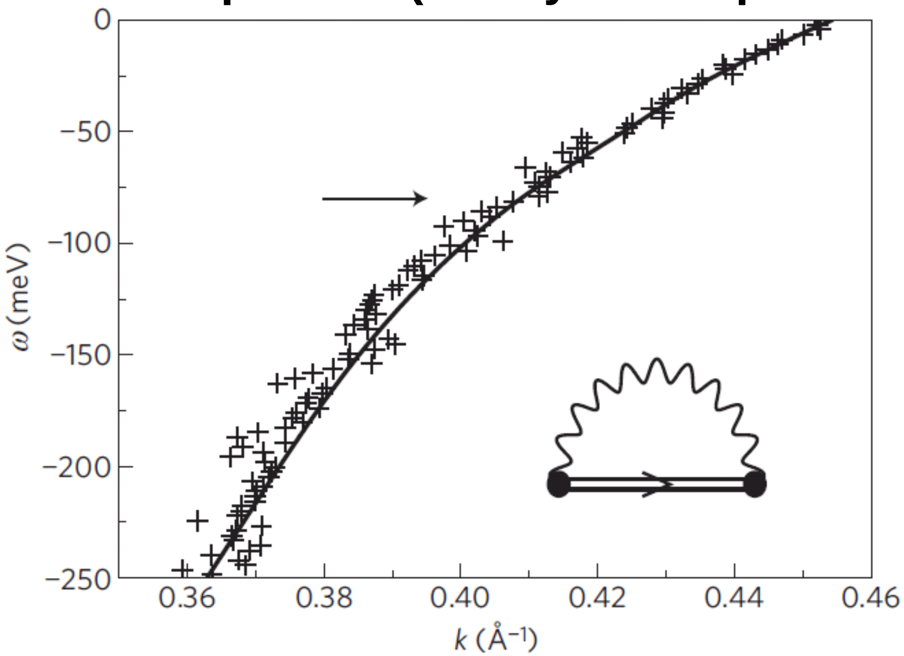
Consistent with kink due to half-breathing phonons

Spin-fluctuation scenario for kinks and pairing

Spin excitation spectrum



Nodal dispersion (theory and experiment)



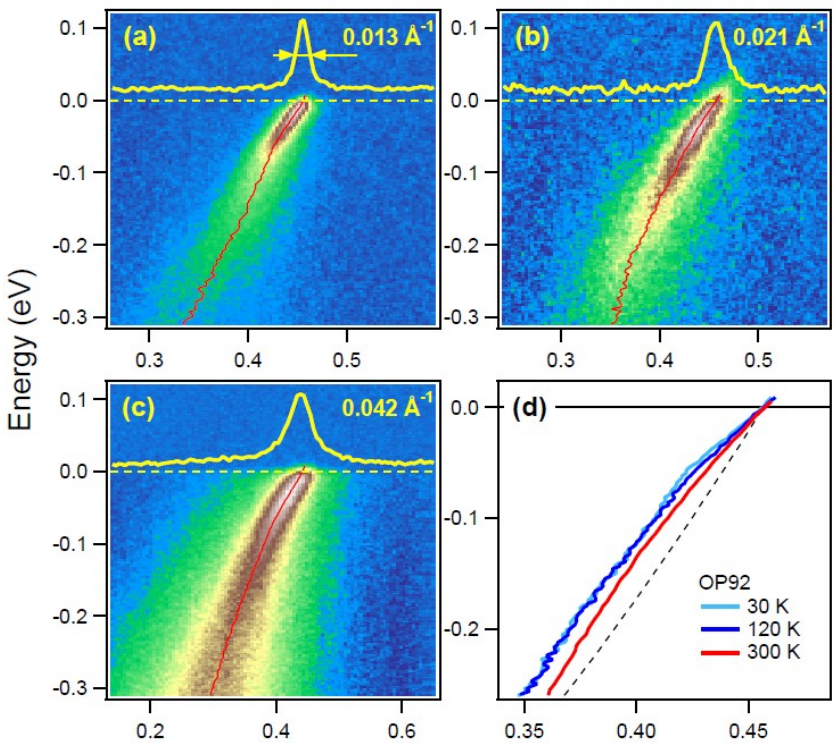
Combination of inelastic neutron scattering and angle resolved photoemission gives consistent picture for paring and kink

Dahm et al., Nature Phys. **5**, 217 (2009)

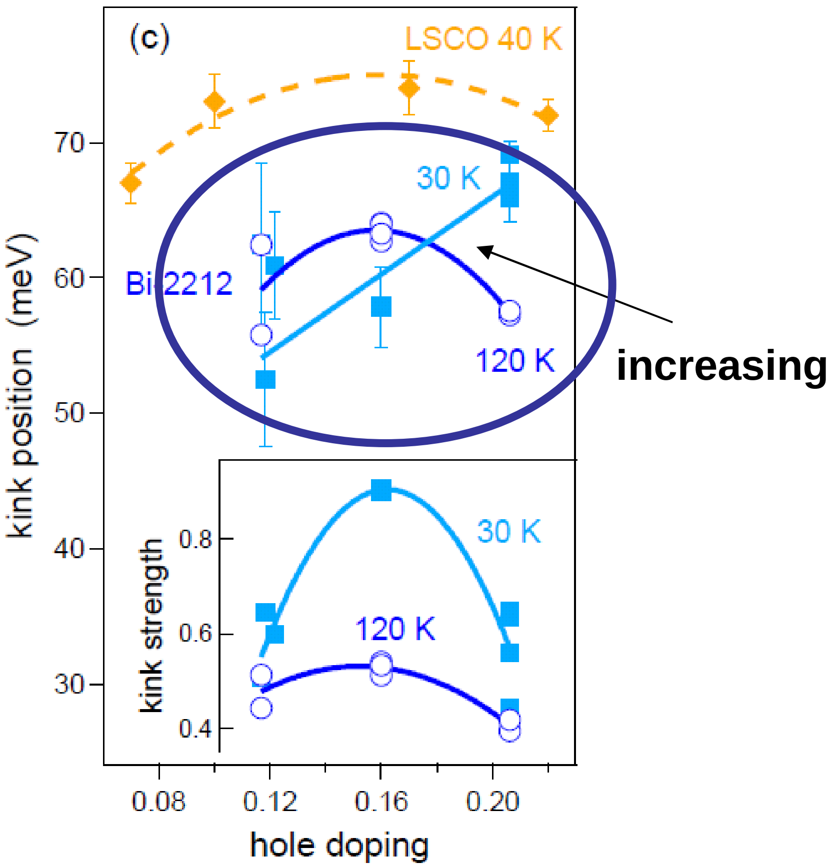
$$V_{\mathrm{eff}}(\mathbf{Q},\Omega) = \frac{3}{2}\,\bar{U}^2\,\chi(\mathbf{Q},\Omega) \qquad \bar{U} = 1.59\,\mathrm{eV}$$

Peculiar doping dependence of kink position

ARPES spectra and analysis



Doping dependence of kink



H. Kordyuk et al, PRL **97**, 017002 (2006)

Argument against phonons - expected to soften or remain constant with doping

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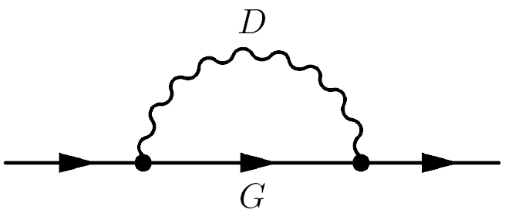
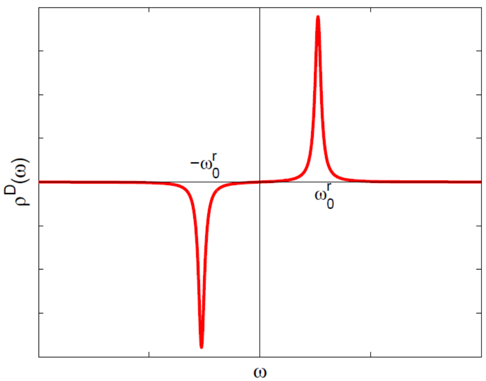
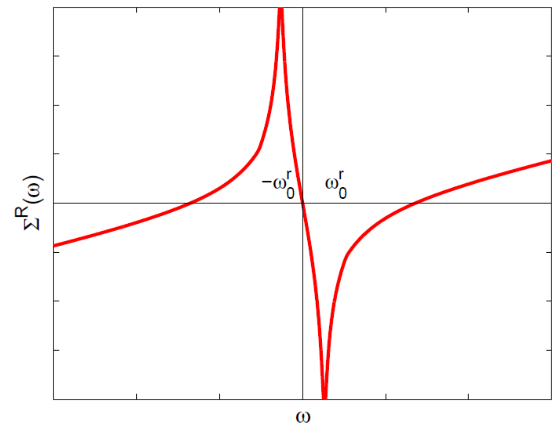
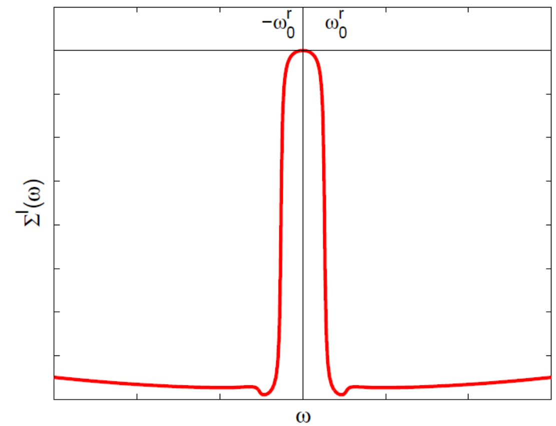
Kinks through coupling to a bosonic mode





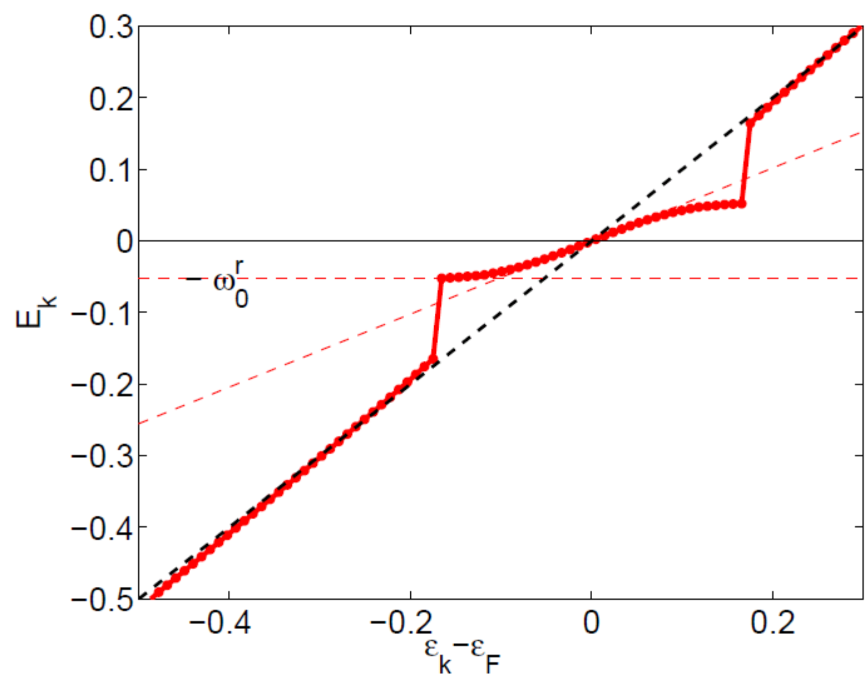
Electronic self energy





Electronic dispersion $G_k(\omega)^{-1} = 0$

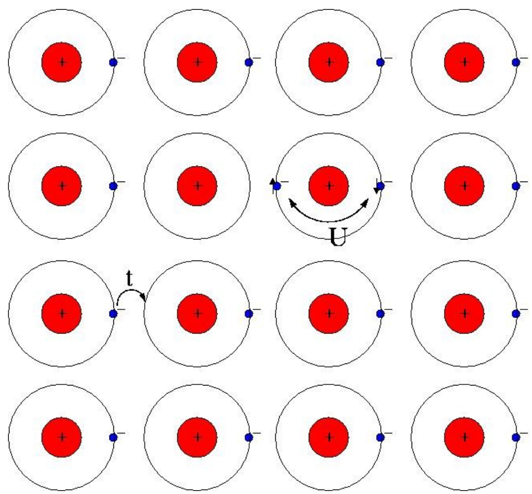
$$E_{\boldsymbol{k}} - \varepsilon_{\boldsymbol{k}} + \mu - \mathrm{Re}\Sigma(E_{\boldsymbol{k}}) = 0$$

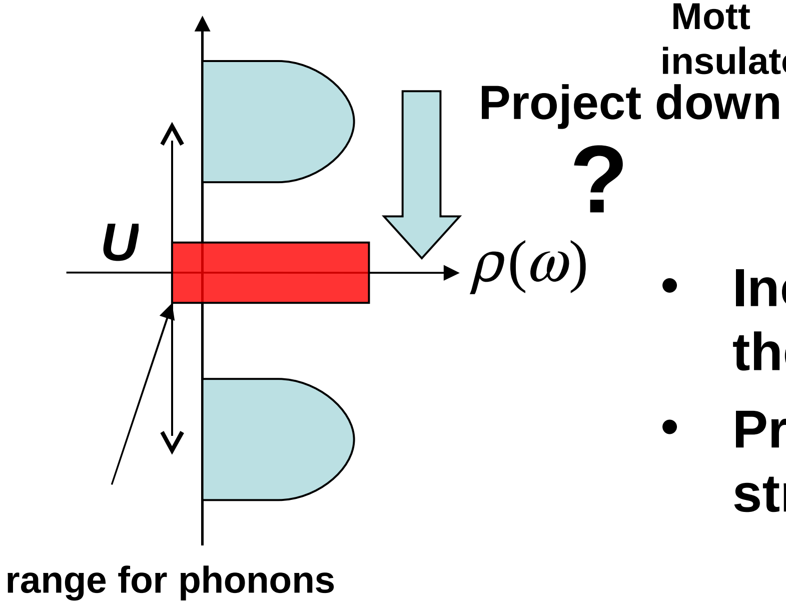


Kink magnitude and position

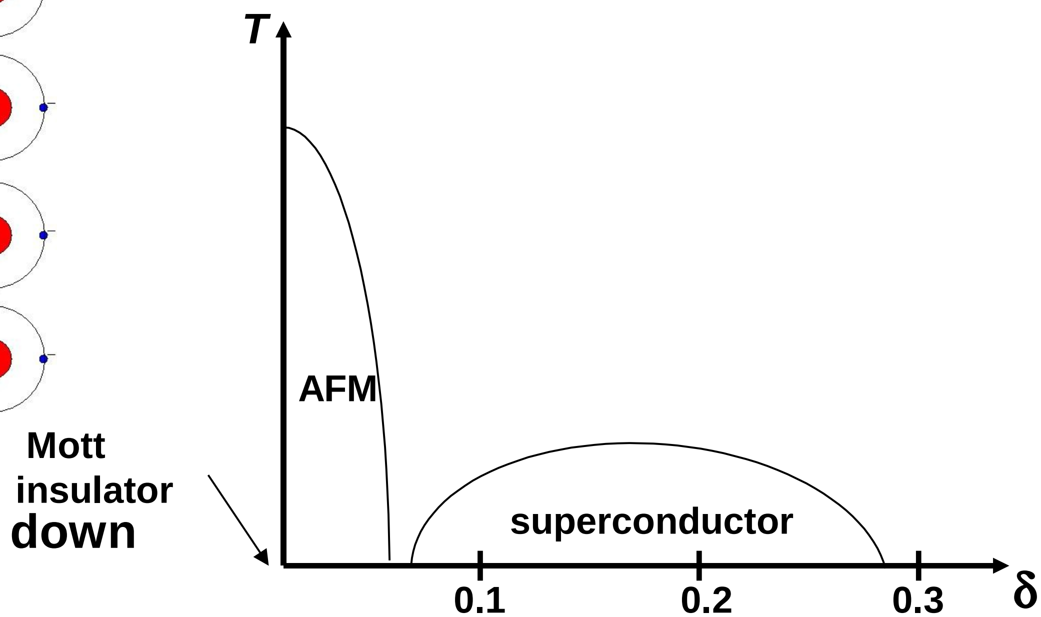
$$\boxed{m_k = 1+\lambda \quad \omega_k \simeq \omega_0^r}$$

Modelling cuprates as doped Mott insulators





schematic phase diagram

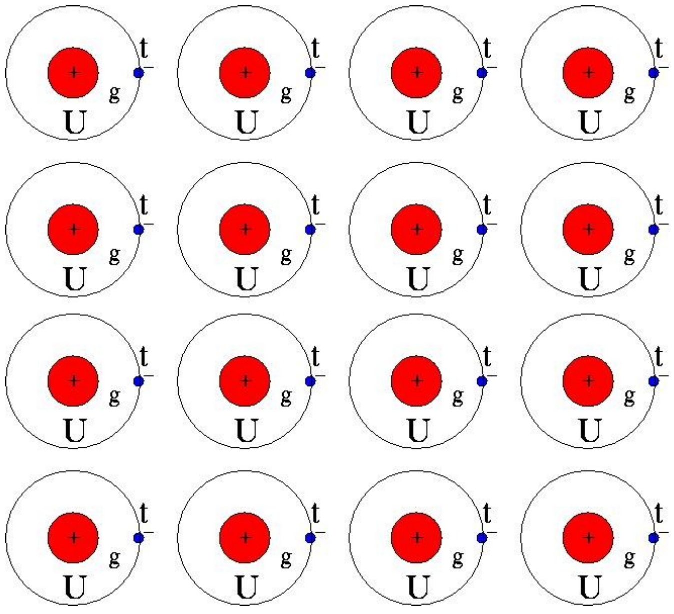


- **Include the strong correlation effects in the description**
- **Projection to the phonon scale is not straight forward**

Hubbard-Holstein (HH) model

$$H = -t \sum_{i,j,\sigma} (c^\dagger_{i,\sigma} c_{j,\sigma} + \mathrm{h.c.}) + U \sum_i \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow}$$
$$+\omega_0 \sum_i b_i^\dagger b_i + g \sum_i (b_i + b_i^\dagger)\Big(\sum_\sigma \hat{n}_{i,\sigma} - 1\Big)$$

Describe coupling to optical phonon in the presence of strong electron-electron repulsion



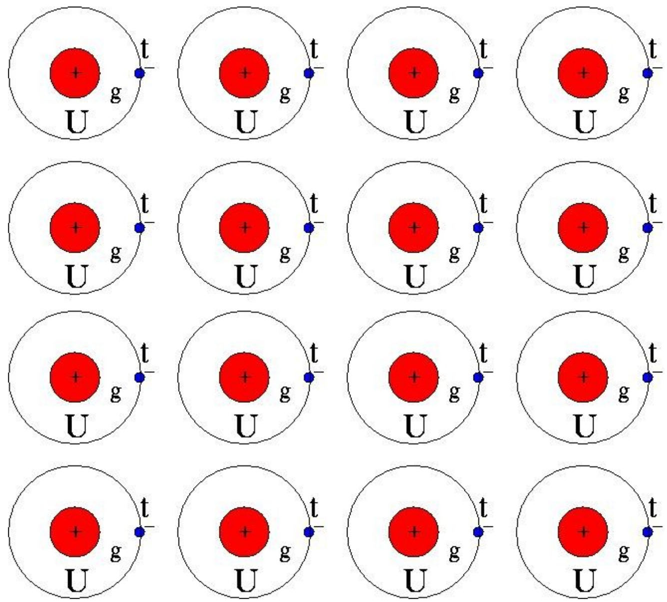
U **and** $\lambda = \rho_0 \frac{2g^2}{\omega_0}$

Electron scale $W = 2D = 4t \quad \rho_0 = \frac{2}{\pi D}$

Phonon scale ω_0 **renormalized** ω_0^r

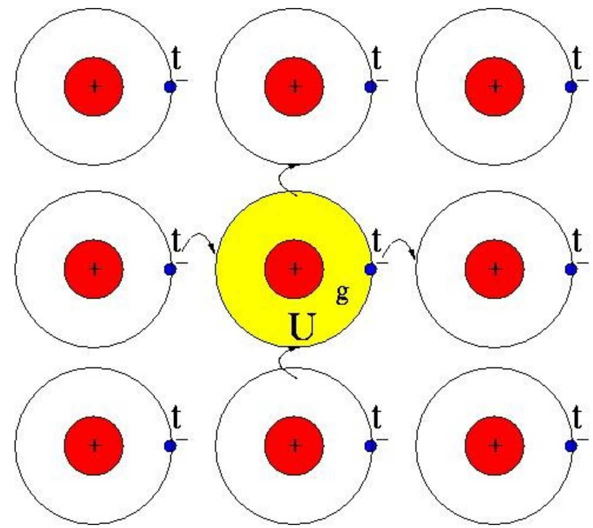
Dynamical mean field theory (DMFT) and Numerical Renormalisation group (NRG)

Hubbard-Holstein model

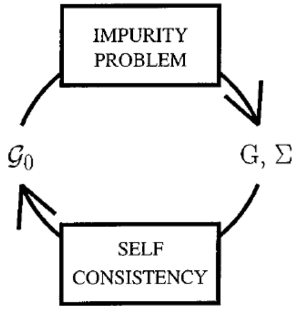


DMFT

Effective Anderson-Holstein model



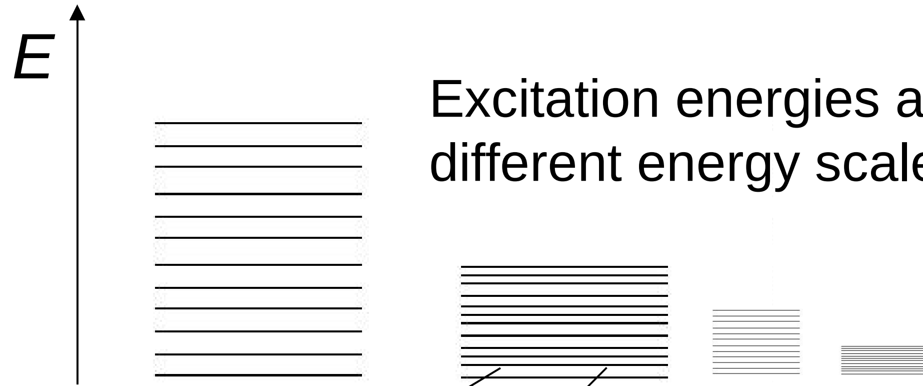
Solve with NRG



Obtain static and dynamic response functions

Dynamic quantities from the NRG

Matrix elements calculated at each NRG step



Excitation energies at different energy scales

$$\rho_{d,\sigma}(\omega) = \frac{1}{Z}\sum_{m,n} |\langle m|c^{\dagger}_{d,\sigma}|n\rangle|^2 \delta(\omega - (E_m - E_n))(\mathrm{e}^{-\beta E_m} + \mathrm{e}^{-\beta E_n})$$

Spectral density of the Green's function in the Lehmann representation

Equations of motion

Higher order Green's functions

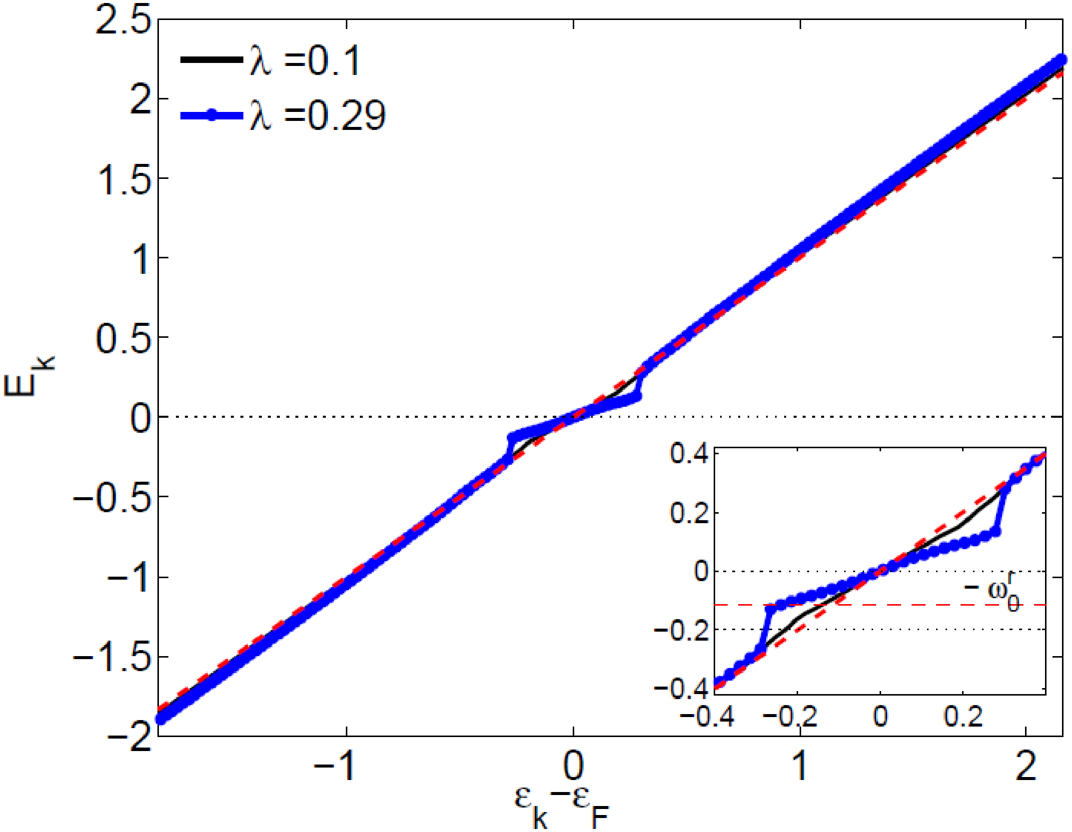
$$\underline{G}_0^{-1}(\omega)\underline{G}(\omega) - U\underline{F}(\omega) - g\underline{M}(\omega) = \mathbb{1}_2$$

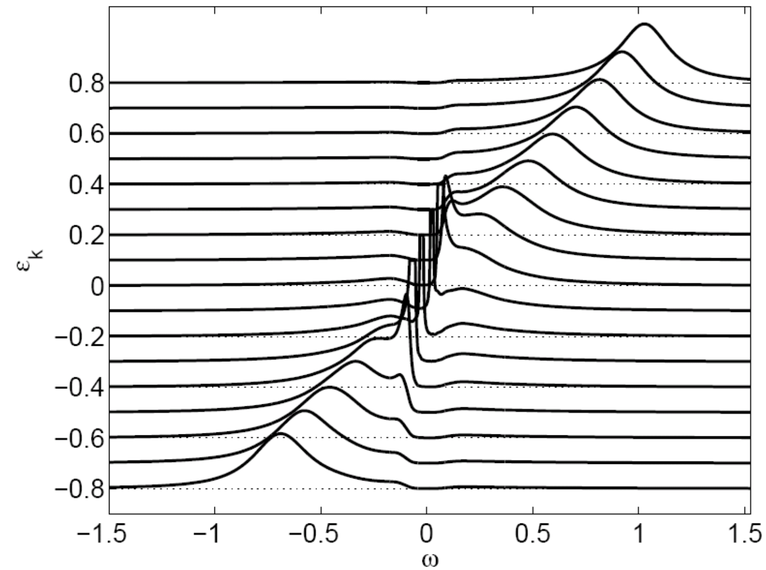
$$\underline{\Sigma}(\omega) = U\underline{F}(\omega)\underline{G}(\omega)^{-1} + g\underline{M}(\omega)\underline{G}(\omega)^{-1}$$

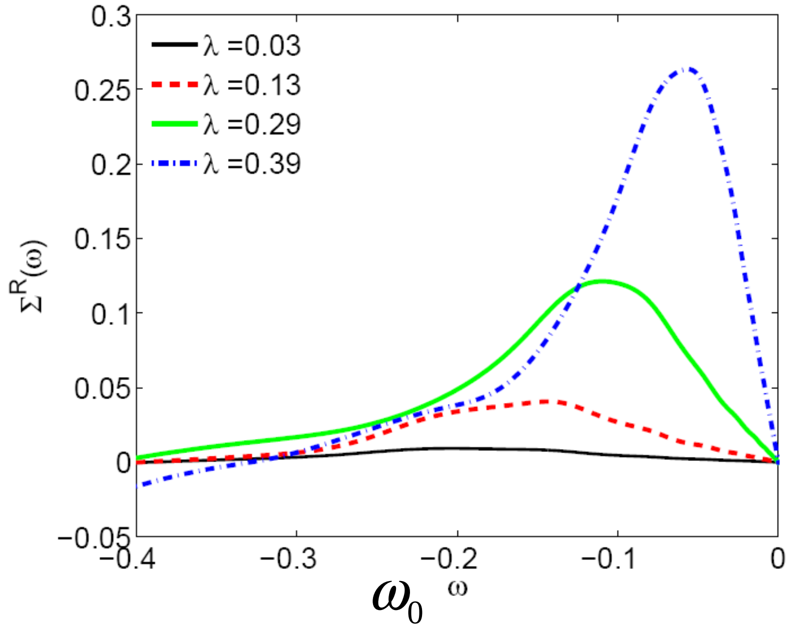
Kinks in DMFT-NRG for U = 0

$\rho_{\boldsymbol{k}}(\omega)$

$\omega_0 / W = 0.05 \qquad n = 0.9$



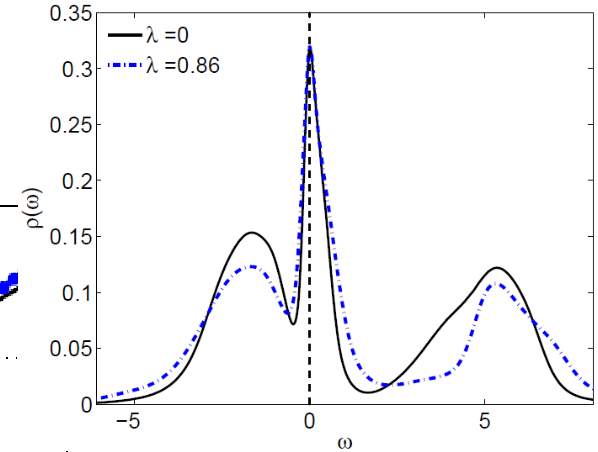




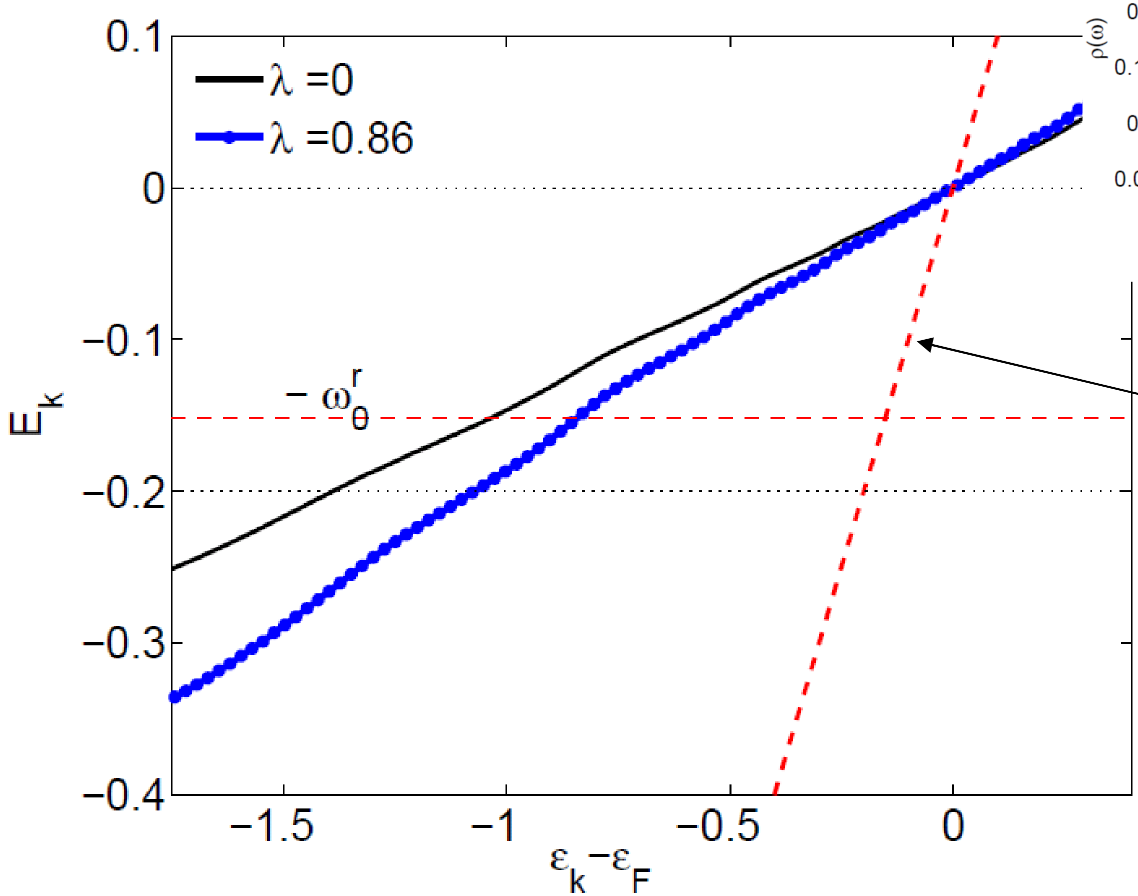
$$\lambda = \rho_0 2g^2/\omega_0$$

Dispersion for large U (PM-DMFT)

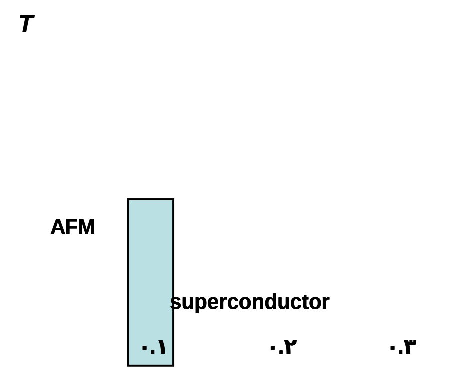
$U/W = 1.5 \qquad \omega_0/W = 0.05 \qquad n = 0.9$



Scenario 1: doped Mott insulator



Bare dispersion

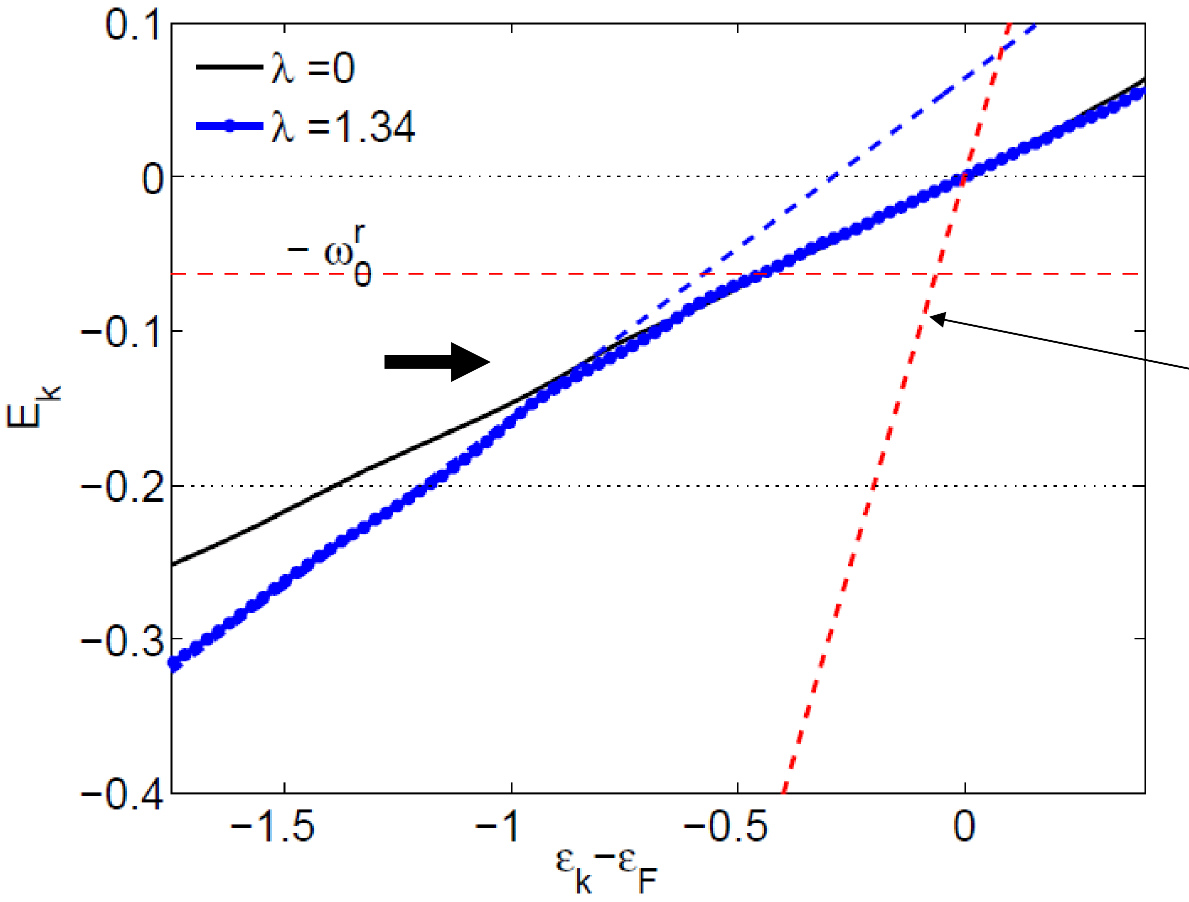


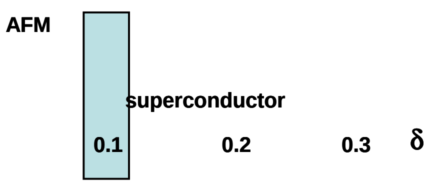
No clear kink visible

Dispersion for large U (PM-DMFT) and larger λ

$U/W = 1.5$ $\omega_0/W = 0.05$ $n = 0.9$

Scenario 1: doped Mott insulator

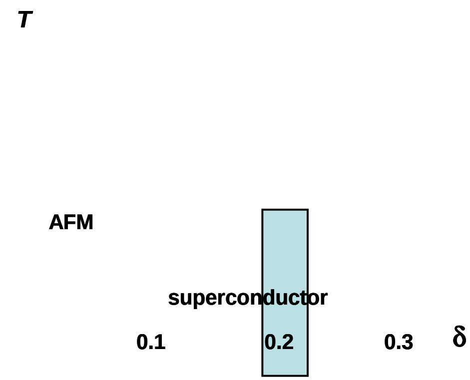




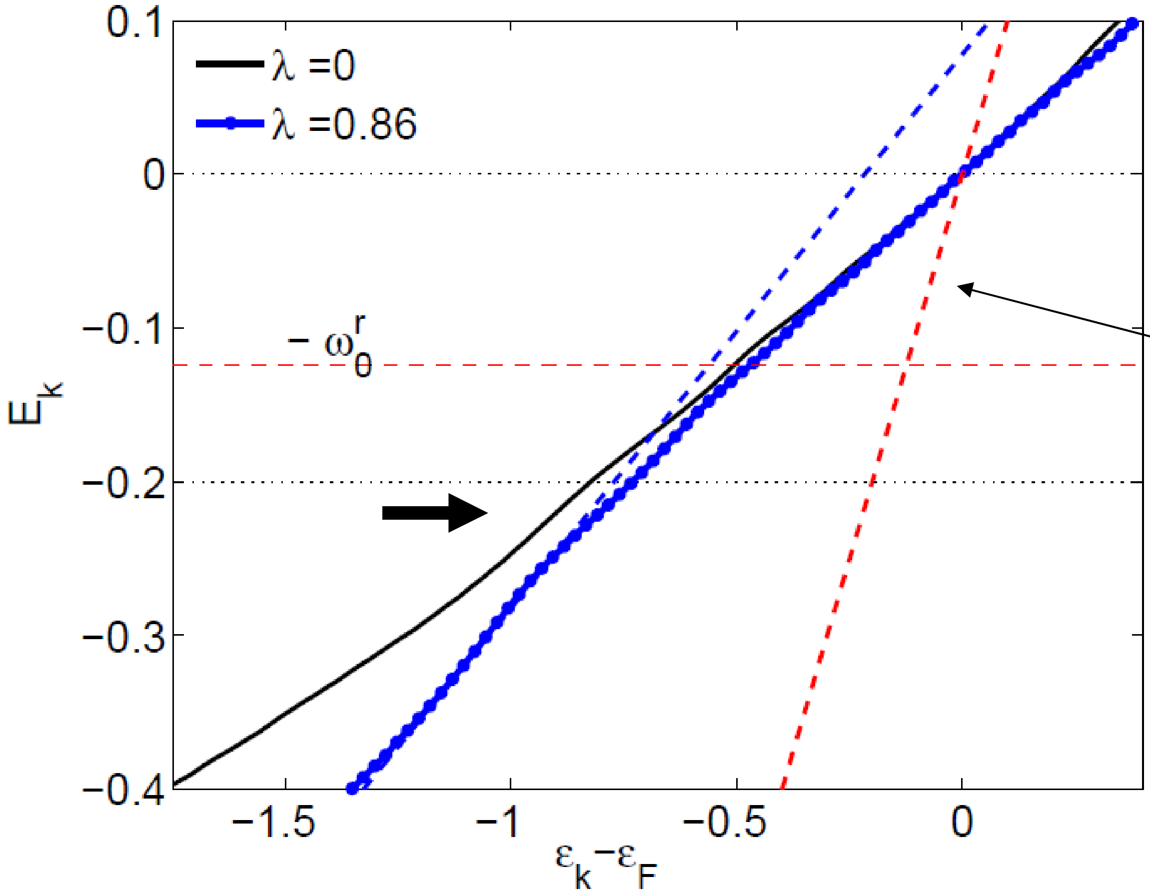
kink visible at higher energy than ω_0^r

Dispersion for large U (PM-DMFT), larger doping

$U/W = 1.5 \qquad \omega_0/W = 0.05 \qquad n = 0.8$



Scenario 1: doped Mott insulator



Bare dispersion

kink visible at higher energy than ω_0^r

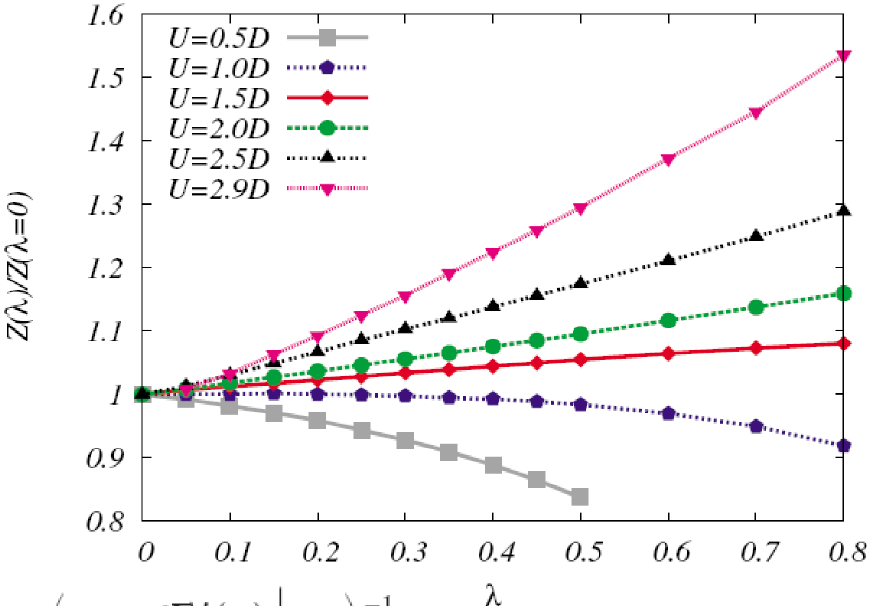
Low energy physics and effective U

Low energy physics can be described by pure Hubbard model with effective *U*

$$\boxed{U_{\text{eff}} = U - \eta\lambda D}$$

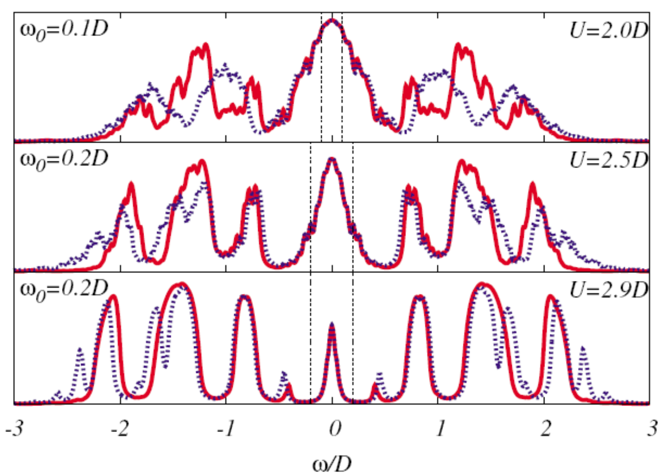
$\eta = 2\omega_0/U/(1 + 2\omega_0/U)$ **is small**

Results at half filling



$$Z = \left(1 - \left.\frac{\partial \Sigma'(\omega)}{\partial \omega}\right|_{\omega=0}\right)^{-1}$$ **Renormalisation factor**

Effect of phonons visible at higher energy

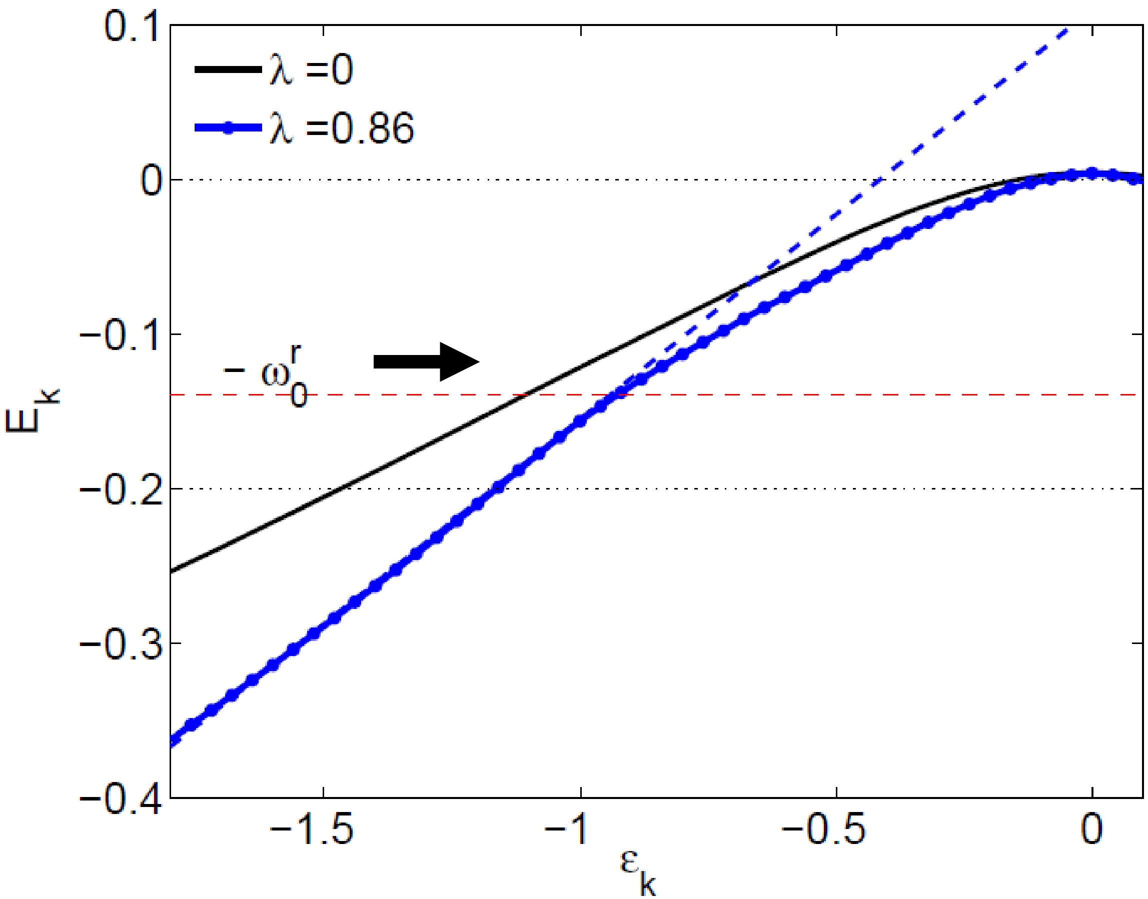


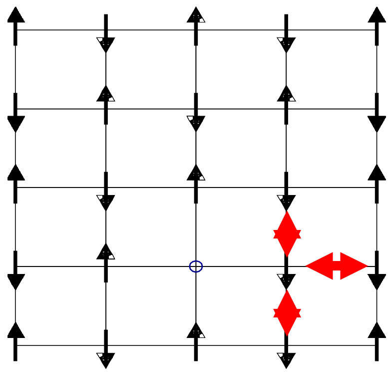
G. Sangiovanni et al., PRL **95**, 226401 (2005), PRB **73**, 165123 (2006)

Dispersion including AFM correlations (AF-DMFT)

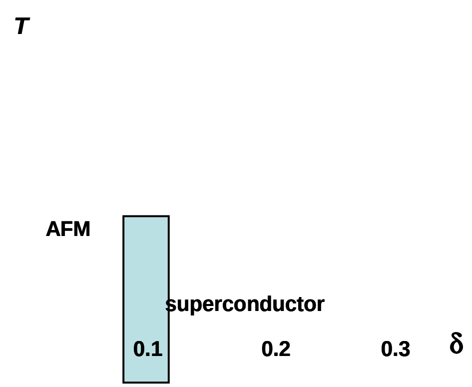
$U/W = 1.5 \qquad \omega_0/W = 0.05 \qquad n = 0.9$

Scenario 2: doped AFM





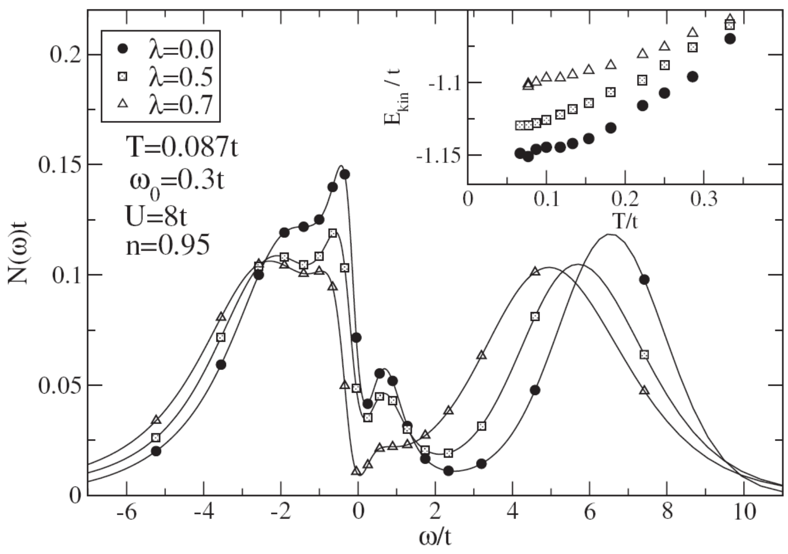
Include antiferromagnetic correlations by imposing AFM order

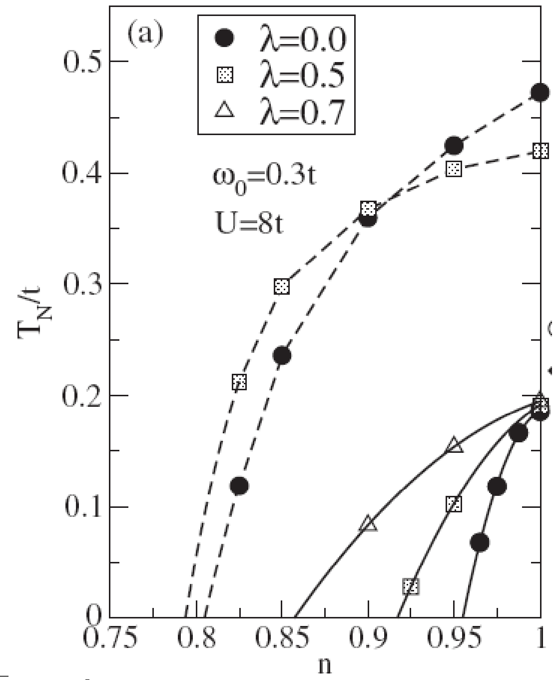


Kink visible at phonon energy ω_0^r

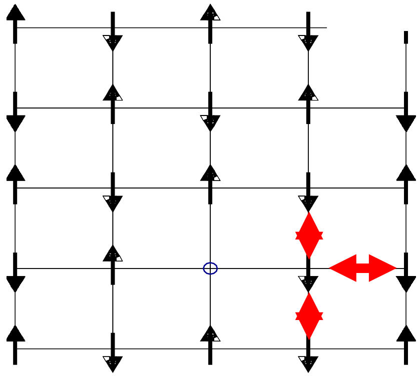
Synergistic polaron formation

- **Polaron formation enhanced by antiferromagnetic correlations**





- **Antiferromagnetic order enhanced by electron phonon coupling**



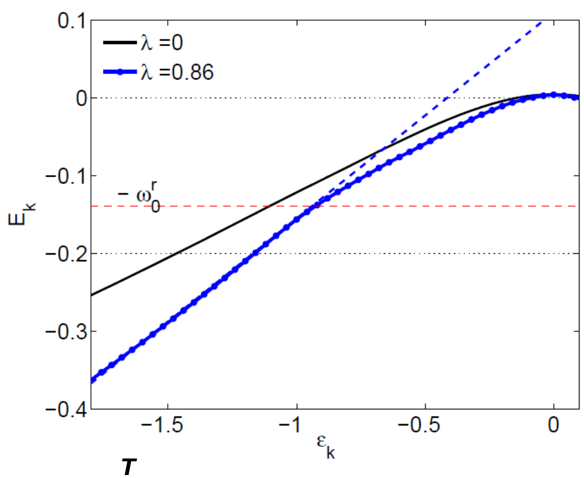
A. Macridin et al, PRL **97**, 056402 (2006)

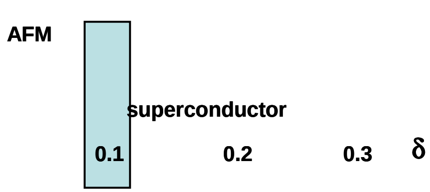
Peculiar doping dependence of kink position

Scenario 2:

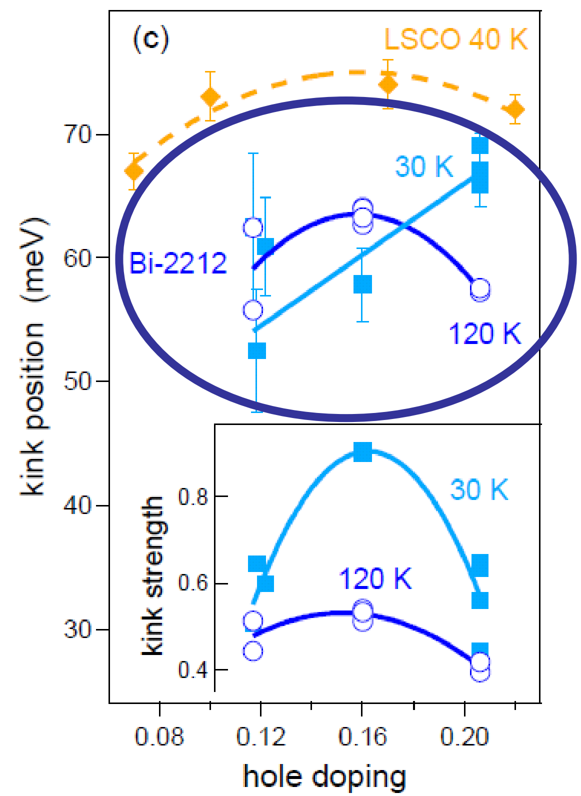
Small doping, large AF correlations

Kink at phonon energy





Doping dependence of kink

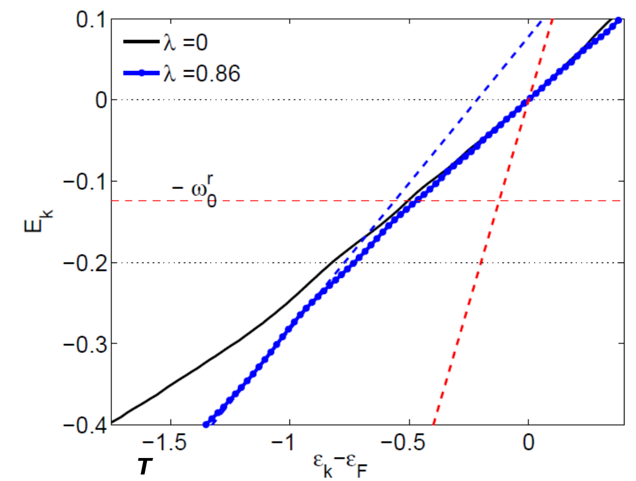


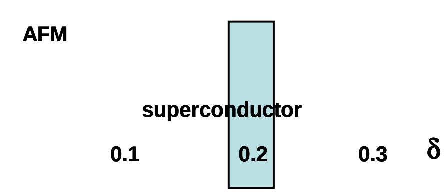
H. Kordyuk et al, PRL **97**, 017002 (2006)

Scenario 1:

Larger doping, less AF correlations

Kink at higher energy





Overview

1. Phase diagram: competing interactions for the Hubbard Holstein model at half filling
   - Normal state
   - With symmetry breaking
2. Kinks in the nodal dispersion of the copper-oxide superconductors
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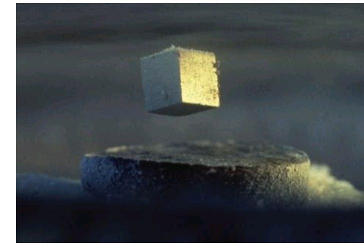
3. Quantitative reliability of Migdal-Eliashberg theory for strong coupling superconductivity
   - Effective coupling
   - Retardation effects

Conventional electron-phonon superconductivity



Transition temperature

$$T_c = \frac{\langle\omega\rangle}{1.2} e^{-\frac{1.04(1+\lambda)}{\lambda-\mu_c^*(1+0.62\lambda)}}$$

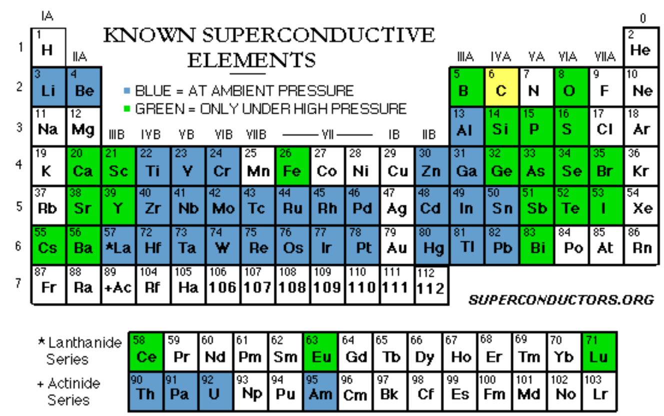
Spectral gap

$$\Delta_{\mathrm{sp}} = 2\omega_0^r e^{-\frac{m^*/m_0}{\lambda-\mu_c^*\left(1+\frac{\lambda}{2Z_2}\right)}}$$

Dimensionless coupling constant

For strong coupling superconductors

$$\lambda \sim 1-3$$



McMillan, Phys. Rev. **167**, 331 (1968)

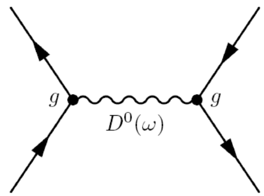
Allen, Dynes, Phys. Rev. B **12**, 905 (1975)

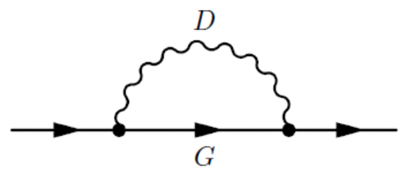
Carbotte, Rev. Mod. Phys. **62**, 1027 (1990)

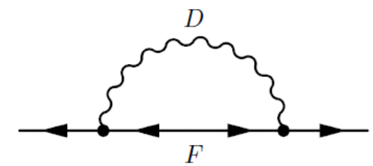
Conventional superconductivity
Migdal-Eliashberg theory

Attraction mediated by phonons

λ Dimensionless effective electron-electron attraction

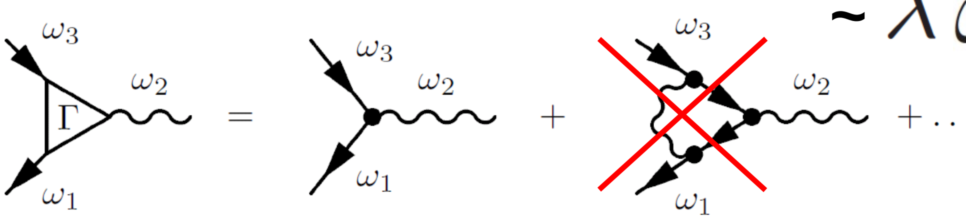






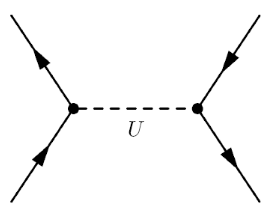
Vertex corrections

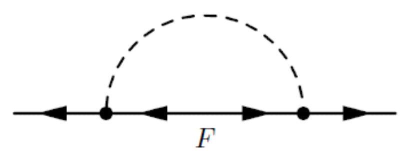
$$\sim \lambda\, \omega_0^r / W$$



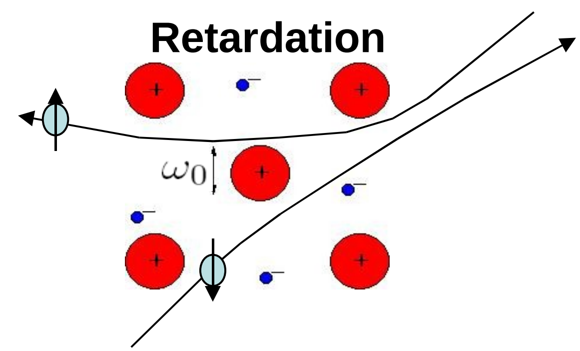
Coulomb repulsion

μ_c Fermi surface average of screened Coulomb repulsion





Retardation



Breakdown of Migdal-Eliashberg theory

- **Model studies going beyond ME diagrammatics**

  P Benedetti, R Zeyher, Phys. Rev. B **58**, 14320 (1998)

  AS Alexandrov, EPL **56**, 92 (2001)

  D Meyer et al., PRL **89**, 196401 (2002)

  M Capone, S Ciuchi, PRL **91**, 186405 (2003)

- **Indication that ME theory breaks down at intermediate coupling strength λ ~ 1 (bipolaronic pairs, metal insulator transition)**

Abstract. – In view of some recent works on the role of vertex corrections in the electron-phonon system, we readdress the important question of the validity of the Migdal-Eliashberg theory. Based on the solution of the Holstein model and $1/\lambda$ strong-coupling expansion, we argue that the standard Feynman-Dyson perturbation theory by Migdal and Eliashberg *with* or without vertex corrections cannot be applied if the electron-phonon coupling is strong ($\lambda \geq 1$) at any ratio of the phonon, ω and Fermi, E_F energies. In the extreme adiabatic limit ($\omega \ll E_F$) of the Holstein model electrons collapse into self-trapped small polarons or bipolarons due to spontaneous translational-symmetry breaking at $\lambda \simeq 1$. With the increasing phonon frequency the region of the applicability of the theory shrinks to lower values of $\lambda < 1$.

Pairing function in conventional superconductivity

Dimensionless coupling constant

$$\boxed{\lambda = 2\int_0^\infty \mathrm{d}\omega\, \frac{\alpha^2 F(\omega)}{\omega}}$$

Pairing function

$$\alpha^2_{\boldsymbol{k},\boldsymbol{k}'} F(\omega) = \rho_0 |g_{\boldsymbol{k},\boldsymbol{k}'}|^2 \rho^D_{\boldsymbol{k}-\boldsymbol{k}'}(\omega)$$

$$\rho^D_0(\omega) = \delta(\omega - \omega_0)$$

Fermi surface average

$$\alpha^2 F(\omega) = \frac{1}{\rho_0^2} \sum_{\boldsymbol{k},\boldsymbol{k}'} \alpha^2_{\boldsymbol{k},\boldsymbol{k}'} F(\omega) \delta(\varepsilon_{\boldsymbol{k}} - \mu) \delta(\varepsilon_{\boldsymbol{k}'} - \mu)$$

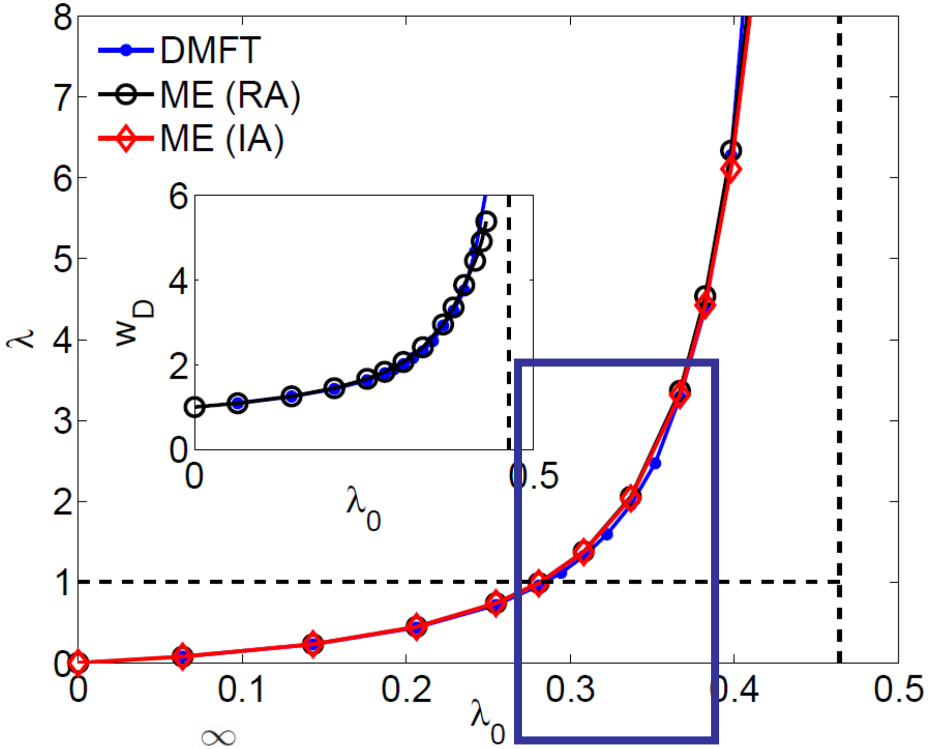
different

$$\Rightarrow \boxed{\lambda = \rho_0 \frac{2g^2}{\omega_0}}$$

$$\boxed{\lambda_0 = \rho_0 2g^2/\omega_0}$$

Phonon renormalization $D(i\omega_m)^{-1} = D^0(i\omega_m)^{-1} - \Sigma_{\mathrm{ph}}(i\omega_m)$

Effective parameters dependence on bare parameters (Holstein model)

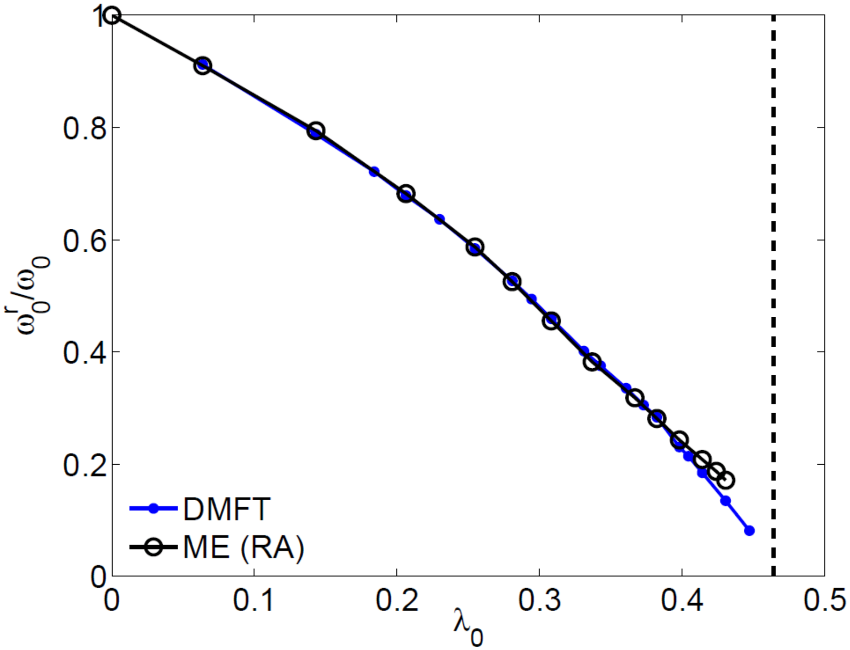


$$w_D = \int_0^\infty \mathrm{d}\omega\, \rho^D(\omega)$$

$$\omega_0^r/\omega_0 = \sqrt{1 - a\lambda_0}$$

$$\lambda = \lambda_0/(1 - a\lambda_0)$$

Phonon softening



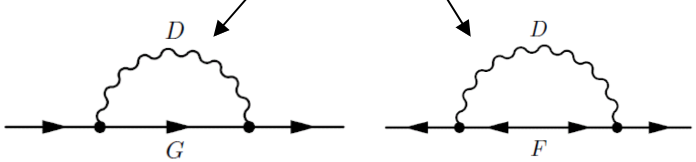
JB, J Han, O Gunnarsson,
cond-mat/1105.2833 (2011)

$$\omega_0 = 0.1t$$

Procedure to test validity for Migdal Eliashberg theory

Standard approach

Experiment, DFT → **Phonons (fully renormalized)**



ME theory

$$T_c\,,\Delta_{\mathrm{sp}}$$

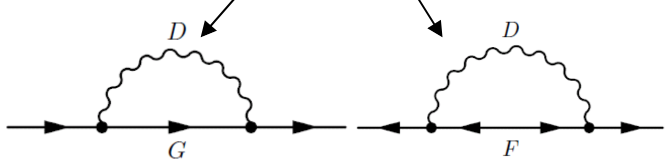
Test for Holstein model in large *d*

DMFT

(1)

Phonons (fully renormalized)

(2)



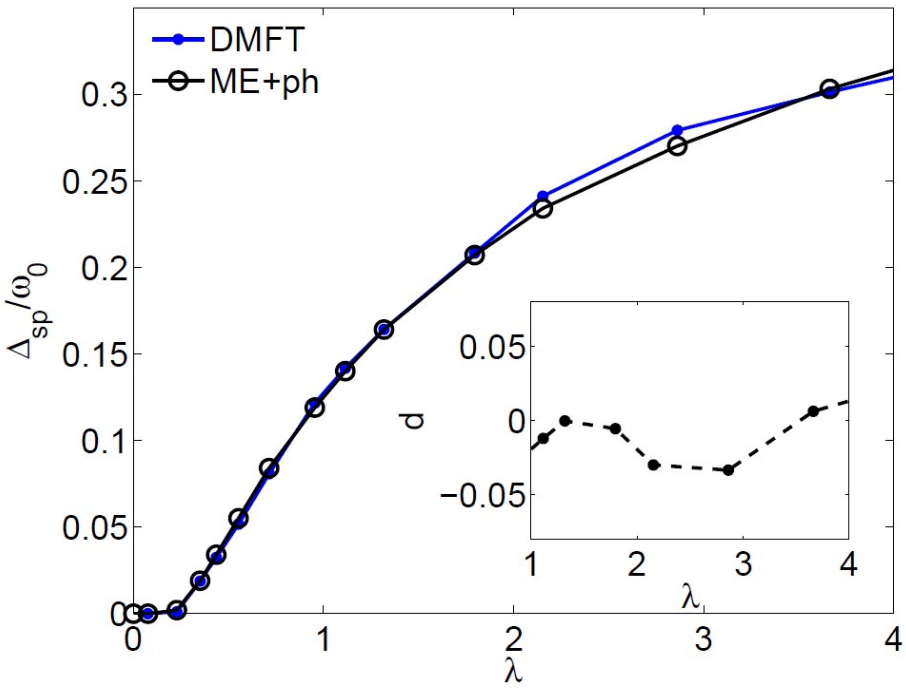
ME+ph theory

$$T_c\,,\Delta_{\mathrm{sp}} \Longleftrightarrow T_c\,,\Delta_{\mathrm{sp}}$$

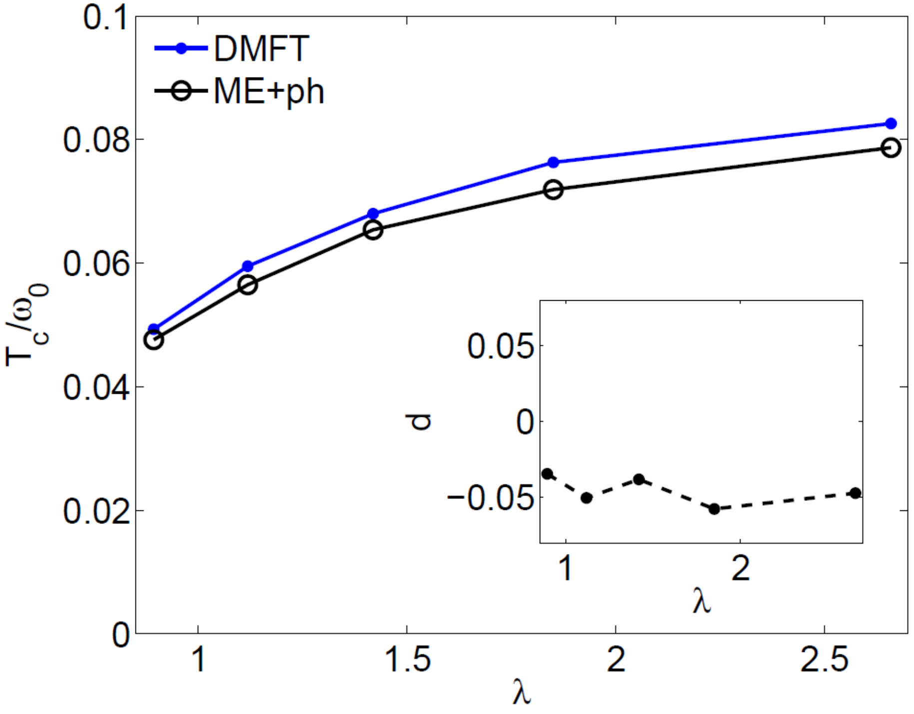
compare

Quantitative test of the reliability of Migdal-Eliashberg theory

Spectral gap at T = 0



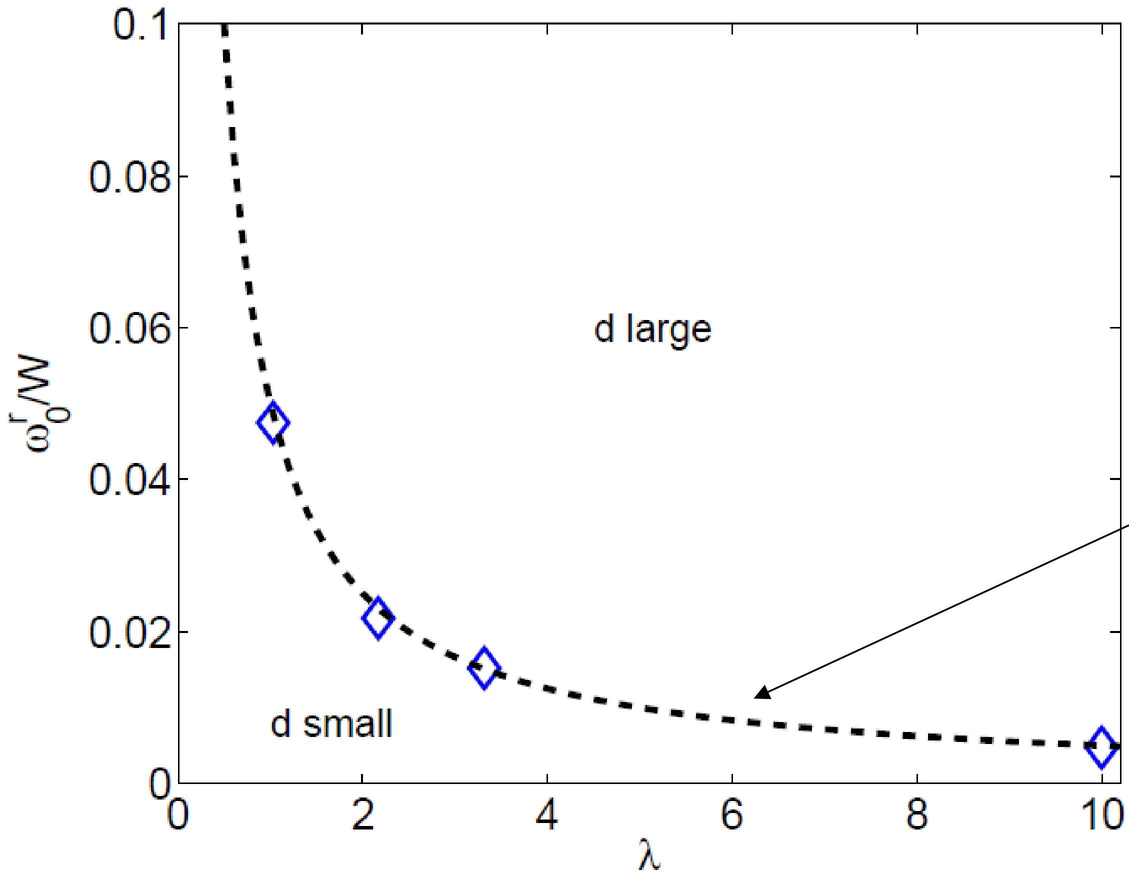
Critical temperature



JB, J Han, O Gunnarsson,
cond-mat/1105.2833 (2011)

$$\omega_0 = 0.1t$$

Quantitative reliability of Migdal-Eliashberg theory



Expansion parameter effective couplings

$$\boxed{\lambda\omega_0^r/W}$$

$$\omega_0^r/W = c/\lambda$$

$$d \simeq 0.1$$

$$d = (\Delta_{\rm sp}^{\rm ME} - \Delta_{\rm sp}^{\rm DMFT})/\Delta_{\rm sp}^{\rm ME}$$

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preliminary results

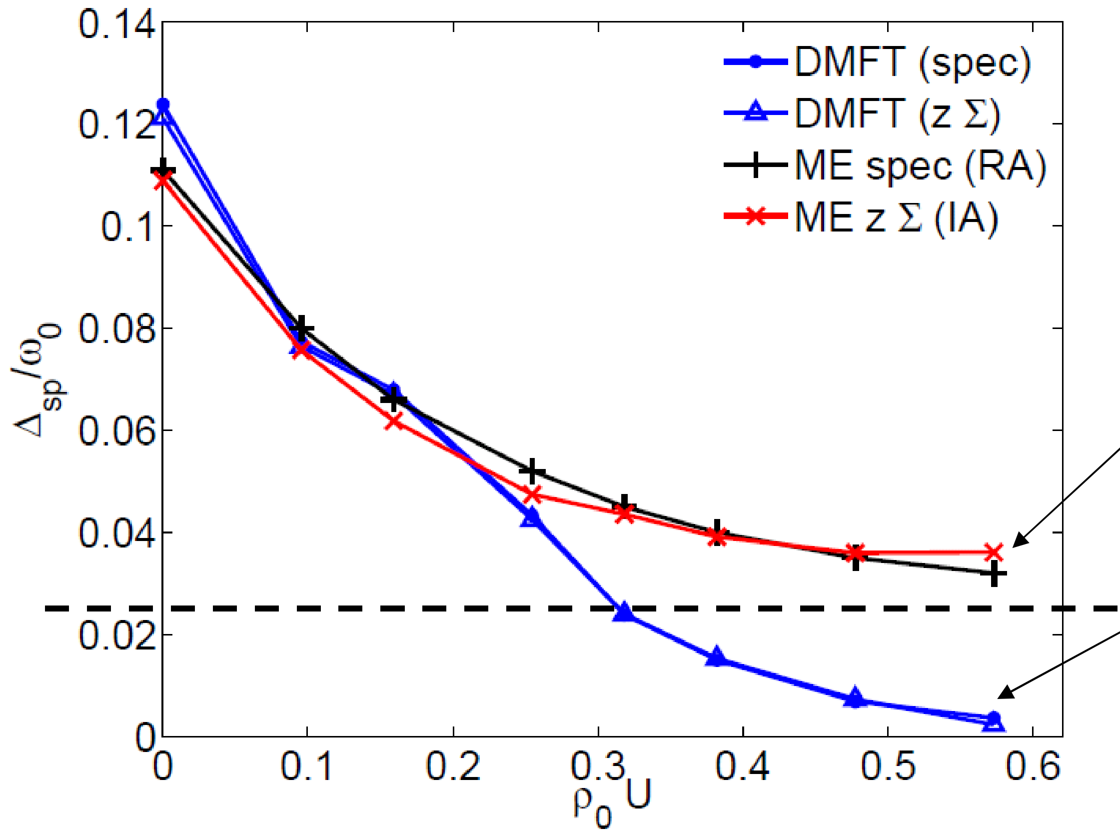
Retardation effects as a function of U

Spectral gap $$\Delta_{sp} = 2\omega_0^r e^{-\frac{m^*/m_0}{\lambda-\mu_c^*}}$$

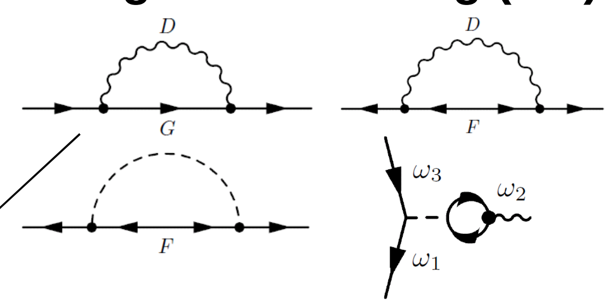
$$\mu_c^* = \frac{\mu_c}{1+\mu_c \log(\frac{D}{\omega_0})}$$

Keep constant

$$\lambda \simeq 1$$



Migdal-Eliashberg (ME)



DMFT superconductivity is stronger suppressed

$$\omega_0^r/D \simeq 0.025 \quad \log(D/\omega_0^r) \simeq 3.68$$

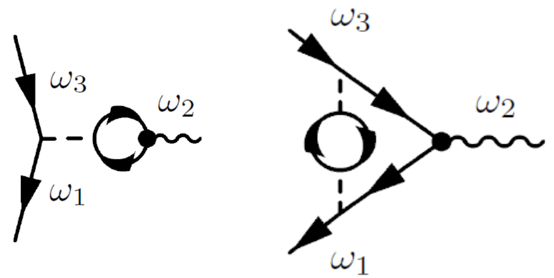
JB, J Han, O Gunnarsson, (2011)

Retardation effects higher order calculation

preliminary results

Terms up to second order

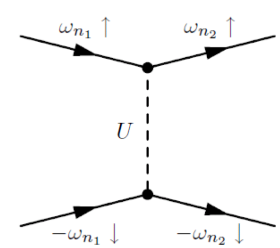
Vertex correction for the electron phonon coupling



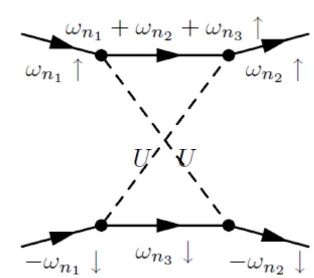
Renormalization of the phonon propagator

Terms up to second order in U

$$\boxed{T_c = \frac{\omega_0^r}{\pi} \exp\left(-\frac{m^*/m_0}{\bar{\lambda} - \bar{\mu}_c^*(1 + c\lambda)}\right)}$$



$$\bar{\mu}_c^* = \frac{\bar{\mu}_c}{1 + \bar{\mu}_c \log(\frac{D}{\omega_0^r})}$$



$$\bar{\lambda} = \lambda(1 - a\mu_c) \qquad \bar{\mu}_c = \mu_c(1 + b\mu_c)$$

JB, J Han, O Gunnarsson, (2011)

preliminary results

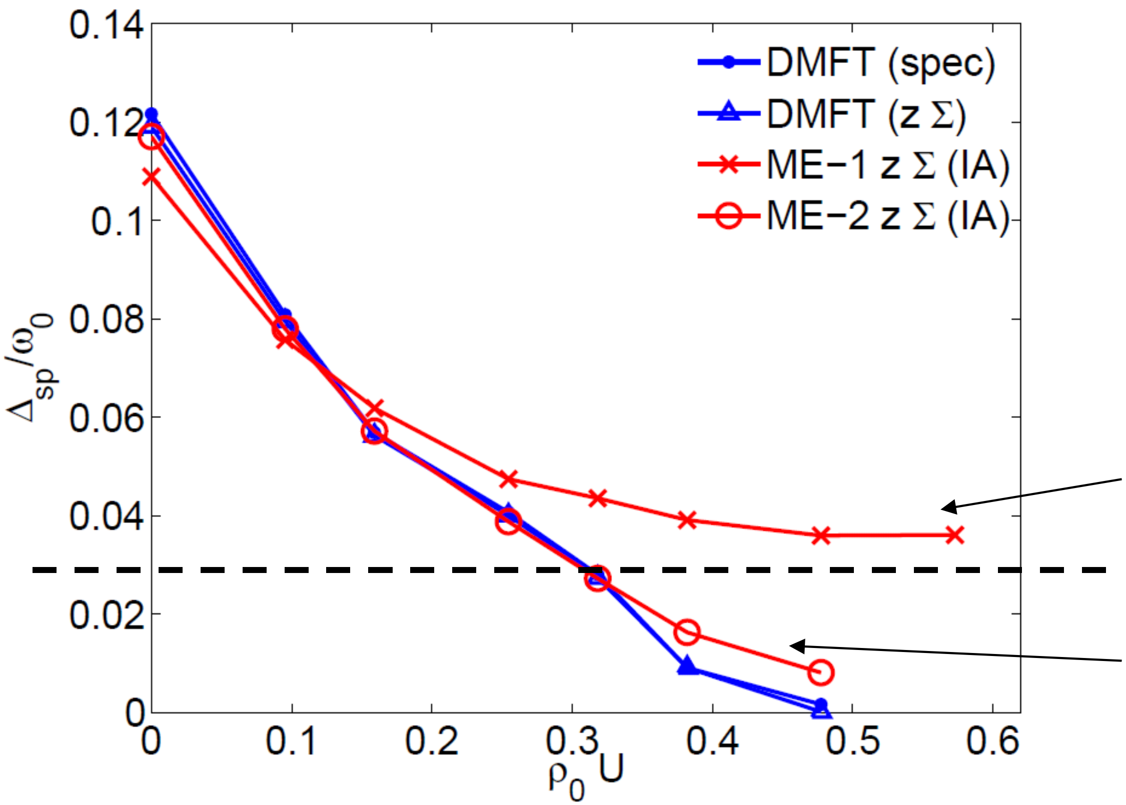
Retardation effects as a function of U

Spectral gap $$\Delta_{\rm sp} = 2\omega_0^r \mathrm{e}^{-\frac{m^*/m_0}{\lambda-\mu_c^*}}$$

$$\mu_c^* = \frac{\mu_c}{1+\mu_c \log(\frac{D}{\omega_0})}$$

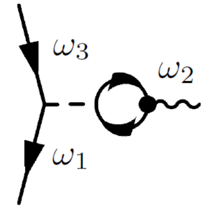
Keep constant

$$\lambda \simeq 1$$



Include vertex correction for the electron-phonon coupling

$$\lambda = \int_0^\infty \mathrm{d}\omega \, \frac{2\alpha^2 F(\omega)}{\omega}$$



$$g^r = g/[1 - U\Pi(0)]$$

Migdal-Eliashberg-1 (ME)

Migdal-Eliashberg-2 (ME)

$$\omega_0^r/D \simeq 0.025 \quad \log(D/\omega_0^r) \simeq 3.68$$

JB, J Han, O Gunnarsson, (2011)

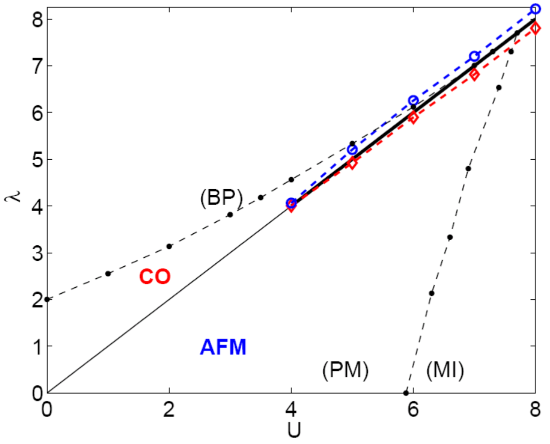
Summary - 1

- **The ground state phase diagram of the Hubbard-Holstein model at half filled for large dimension shows antiferromagnetic and charge order**

- **The transition line is approximately where the bare effective interaction vanishes**

- **We find continuous transitions for smaller coupling and larger phonon frequency and discontinuous transitions for larger coupling and smaller phonon frequency**

- **Find similarities, but also differences with low dimensional results**



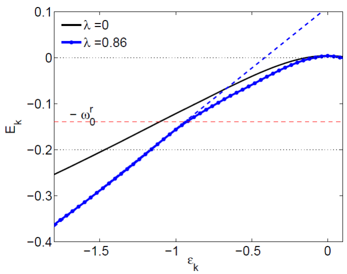
JB, EPL **90**, 27001(2010)

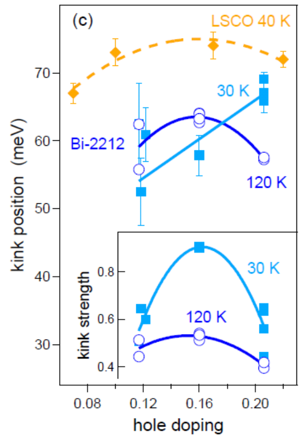
JB, AC Hewson, Phys. Rev. B **81**, 235113 (2010)

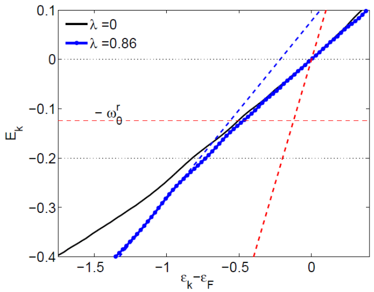
AC Hewson, JB, J. Phys. Cond. Mat. **22**, 155602 (2010)

Summary - 2

- **From ARPES, in cuprates kink positions in the nodal dispersion show a strong dependence on doping**
- **Scenario1: DMFT-NRG calculations in PM state with large U we find kink energy larger than phonon energy; understood with effective interaction U changed little by the phonons**
- **Scenario 2: DMFT-NRG calculations in AF state with large U give kink at phonon energy for comparatively smaller coupling due to cooperative effect**
- **These scenarios can provide the basis for a situation with similar trends as in experiment**







JB, G Sangiovanni, Phys. Rev. B **82**, 184535 (2010)

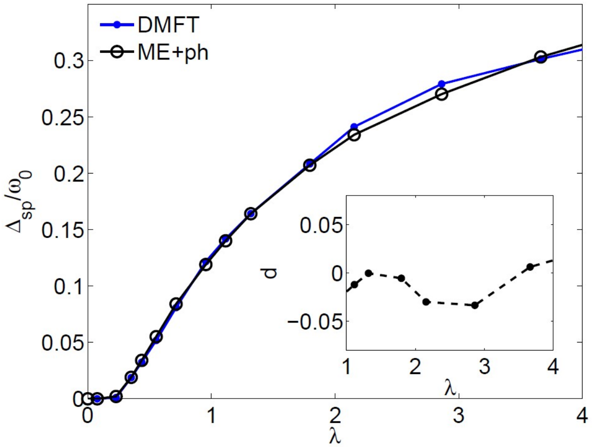
Summary -3

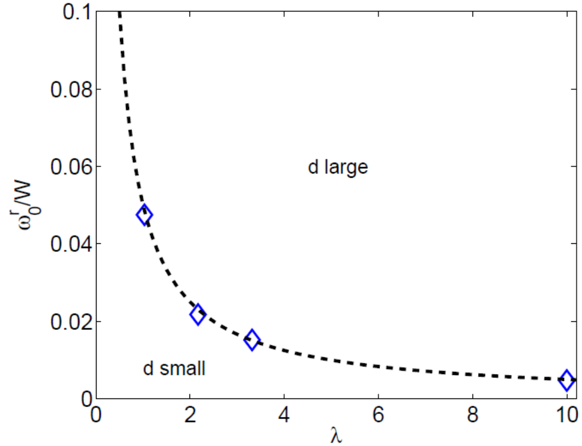
- **We distinguish bare and effective parameters in the electron phonon coupling problem**

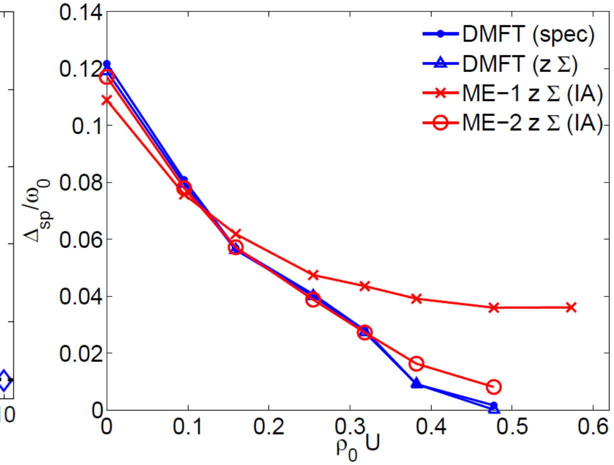
- **Migdal-Eliashberg theory with reliable phonon input is accurate for large couplings if effective phonon scale is small**

- **Anderson-Morel retardation effects and pseudopotential effect well defined for weak coupling and large band width**

- **Modification due to higher order effects expected**







JB, J Han, O Gunnarsson,
cond-mat/1105.2833 (2011)

Thank you for your attention !