Magnetization plateaus in frustrated quantum magnets

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Collaborators

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Outline

- Geometric frustration and novel magnetic phenomena.
- $SrCu_2(BO_3)_2$ and the Shastry-Sutherland Model.
- Field-induced magnetization plateaus.
- Chen-Simons theory.
- New family of Shastry-Sutherland compounds and the generalized Shastry-Sutherland model.

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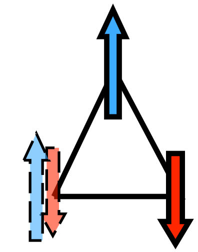
Geometric frustration

Geometric frustration

- ❑ Not all interactions are simultaneously minimized
- ❑ Key element in the study of quantum magnets
- ❑ Rich variety of phases



Spins with AFM interactions

Novel phases and phenomena induced by geometric frustration

- ✓ Dimensional reduction at a quantum critical point in Han Purple
- ✓ Spin supersolid phase in Heisenberg antiferromagnets
- ✓ Spin liquid phases

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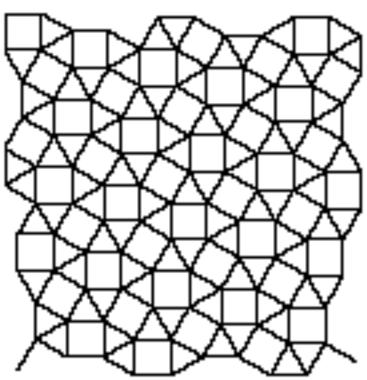
Shastry-Sutherland Lattice

<u>Archimedes, circa. 250 BC:</u>

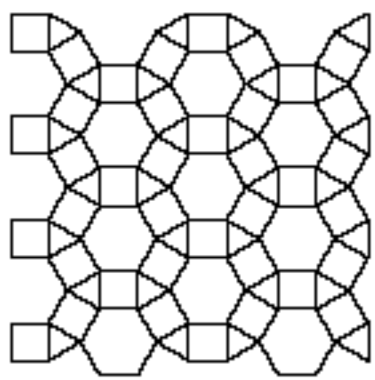
Thought of tiling the 2D plane with symmetric polygons

<u>Johannes Kepler, 1619 AD:</u>

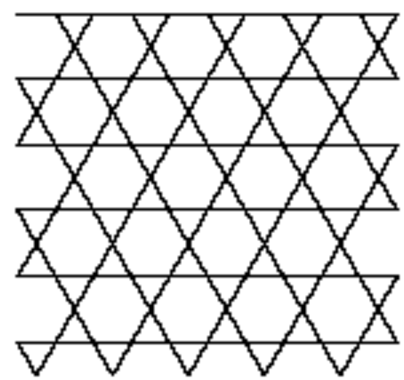
Listed the different Archimedean lattices



T6: $3^2.4.3.4$ = SrCuBO



T7: 3.4.6.4 = bounce



T8: 3.6.3.6 = kagomé

Lattice of interest

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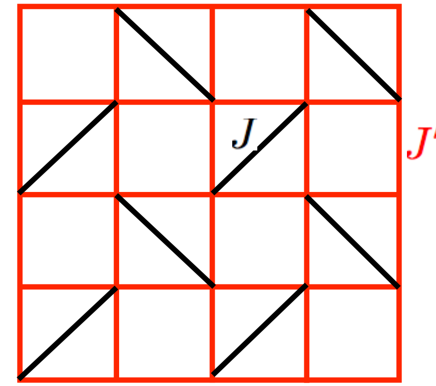
Shastry-Sutherland Model

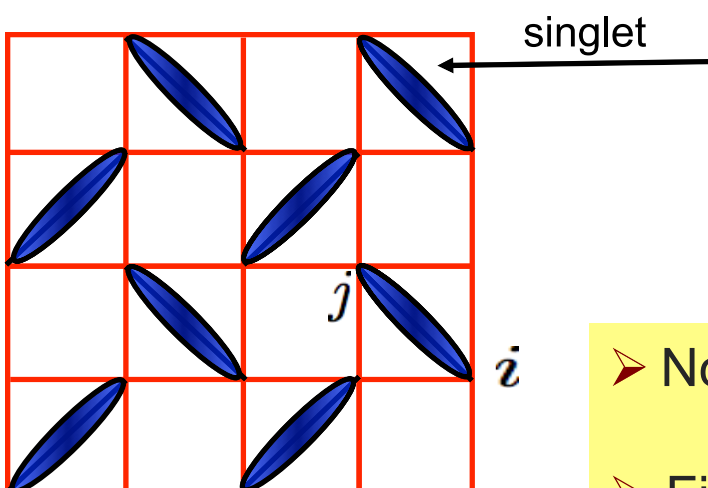
$$H = \sum_{i,j} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_\mathbf{j} - B\sum_i S_i^z$$

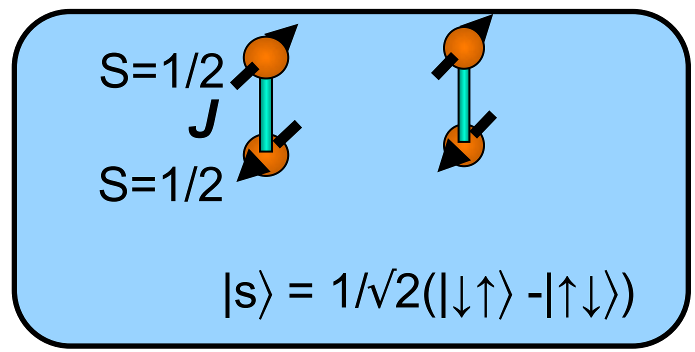
S=1/2 spins with Heisenberg AFM interactions

Exact ground state for J/J’ > 1.35:



Ground state is a direct product of singlets on the diagonals





- Non-magnetic ground state
- Finite gap to lowest (magnetic) excitations

B. Sutherland & B. Shastry, *Physica* **108B** 1069-1070 (1981).

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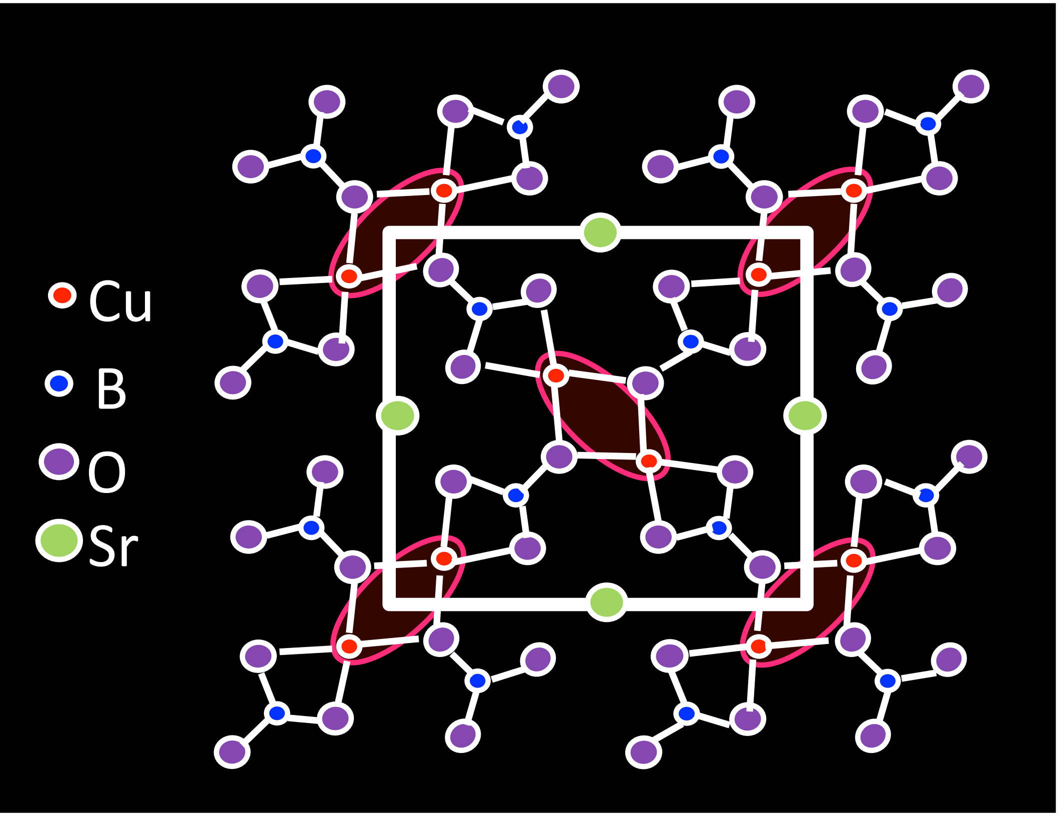
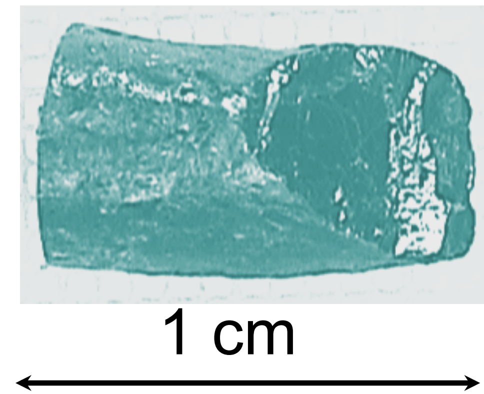
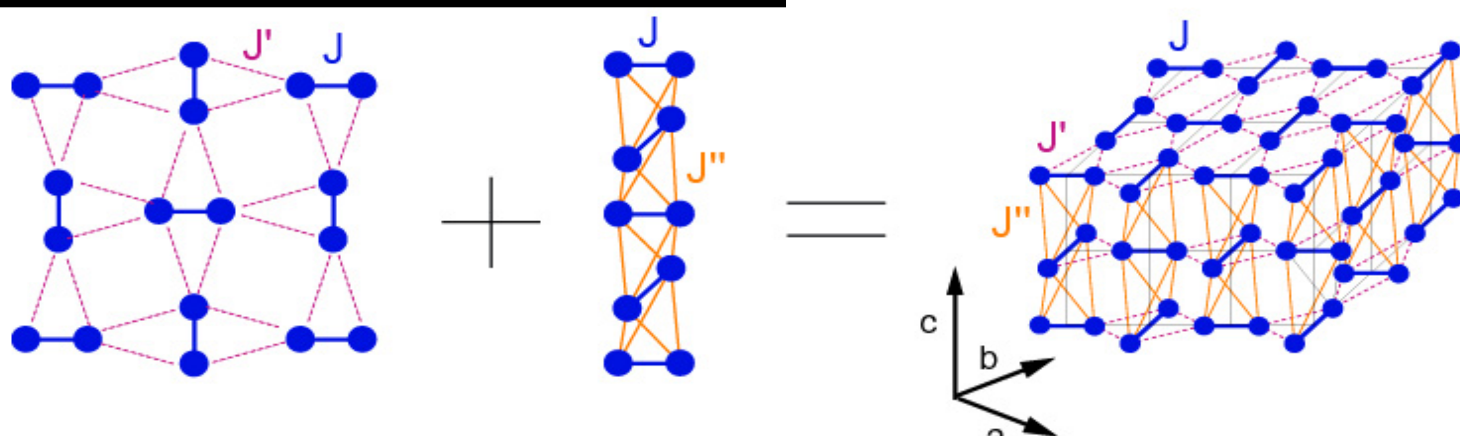
$SrCu_2(BO_3)_2$



R. W. Smith and D. A. Keszler, J. Solid State Chem. **93,** 430 (1991).





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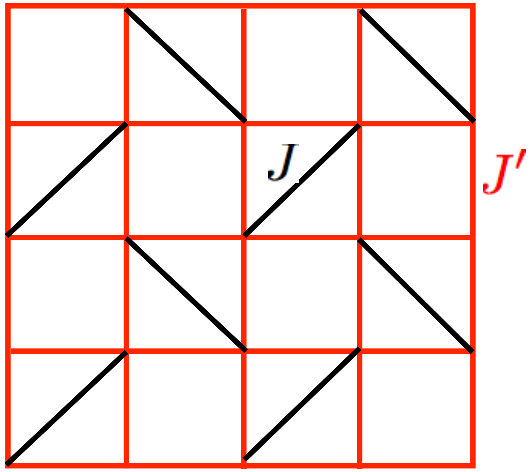
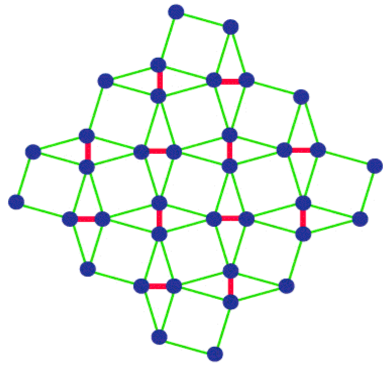
$SrCu_2(BO_3)_2$: SSL magnetic lattice

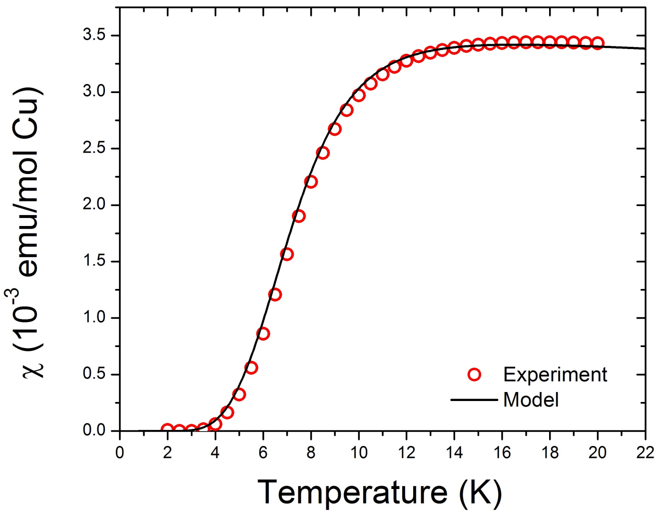


Topologically equivalent



$SrCu_2(BO_3)_2$: Cu^{2+} ions arranged in a SSL lattice

Uniform susceptibility:



ctivated behavior $\Longrightarrow$ spin-gapped ground state

Estimated parametrs:

$$J = 74K \quad J' = 0.62J \quad J/J' \simeq 1.62$$

Consistent with a singlet ground state

Dzyaloshinsky-Moriya interaction

G. A. Jorge, et. al., Phys. Rev. B **71**, 092403 (2005)

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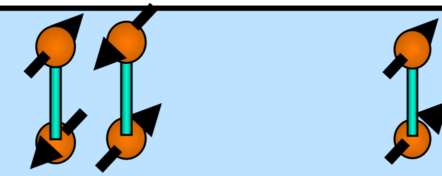
The SSL model in an applied field

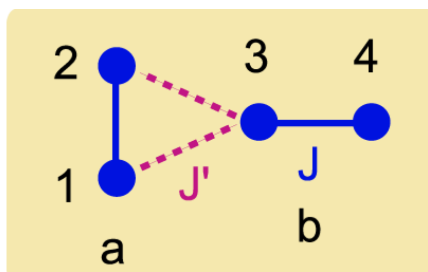
Lowest energy excitations: singlet ⟹ triplet



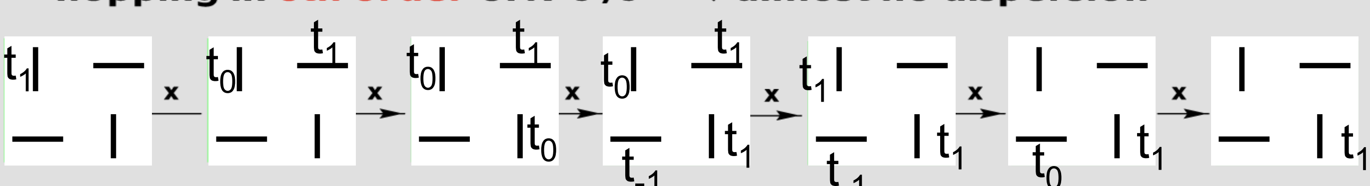
$|s\rangle = 1/\sqrt{2}(|\downarrow\uparrow\rangle - |\uparrow\downarrow\rangle)$ $|t_+\rangle = |\uparrow\uparrow\rangle$

- At sufficiently strong fields, the system has finite density of triplets ⟹ finite magnetization, M(H)
- Because of frustration, the triplets are extremely immobile

Single triplet hopping: 6th order in J'/J



hopping in 6th order of x=J'/J ⇒ almost no dispersion



Miyahara & Ueda, PRL 82, 3701 (1999).

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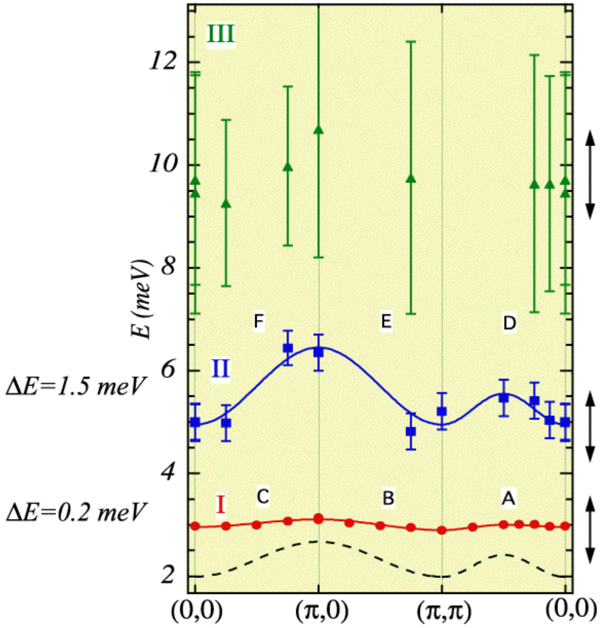
Experimental signatures

- ❑ Flat dispersion in inelastic neutron scattering

- ❑ Novel magnetization plateaus; commensuration energy gain from regular pattern of triplets

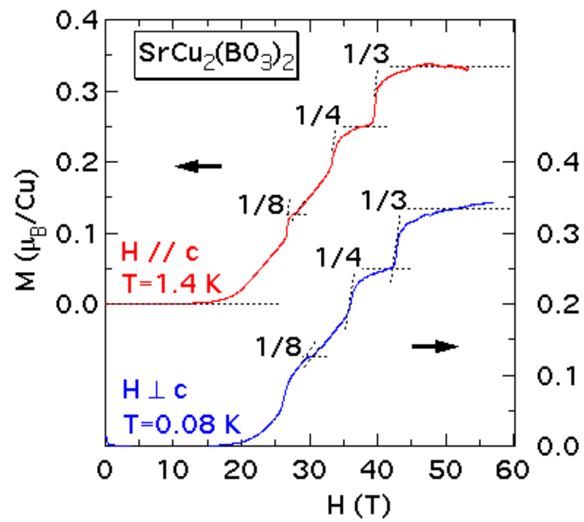
INS



Bound state of two triplets

Quasi flat single triplet mode.

M vs. H



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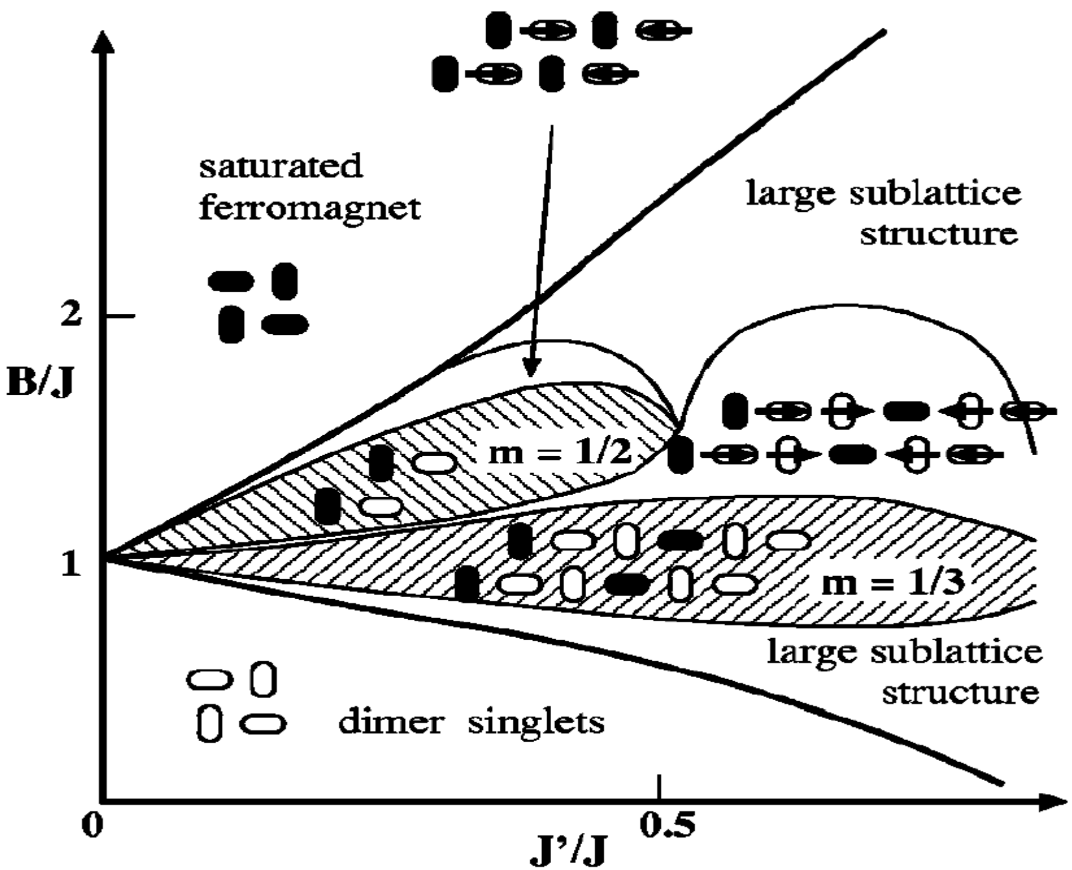
Previous Results

Treat triplets as a gas of classical particles



Cannot capture any longer range density modulation

Keep only the S^z=1 triplets and the effective „repulsive" terms to 3rd order in J'/J

T. Momoi and K. Tosuka, PRB 62, 15067 (2000).

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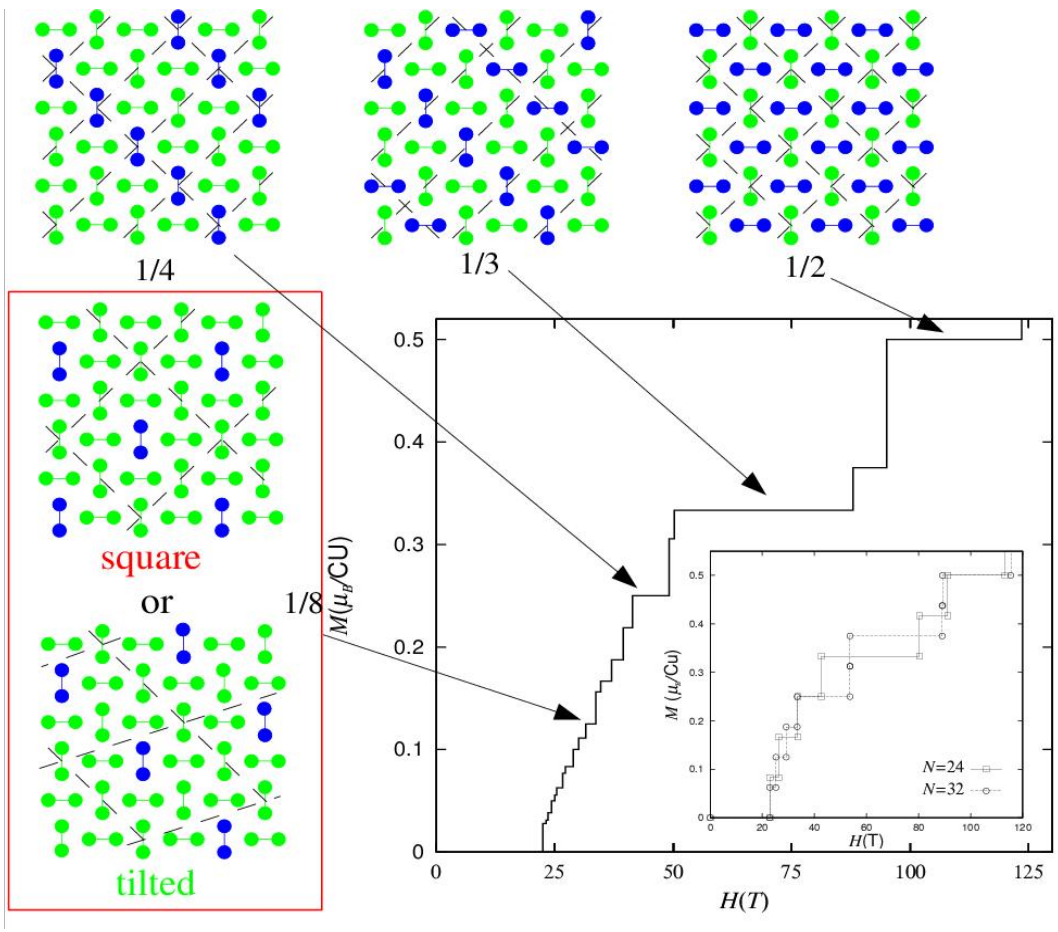
Previous Results (contd.)



$$V(r) = V_n(V_n') \quad \text{if} \quad n = 1, 2, 3$$

$$V(r) = V_0 \frac{e^{-r/\xi}}{r} \quad \text{for others}$$

N_d=16,20,26,32,36 for square unit cells and N_d=24,32 for rectangular cells.

Miyahara & Ueda: J. Phys.: Condens. Matter **15** R327 (2003)

Chern Simons theories

Misguich et al, PRL 87, 097203(2001)

Chern Simons Theory

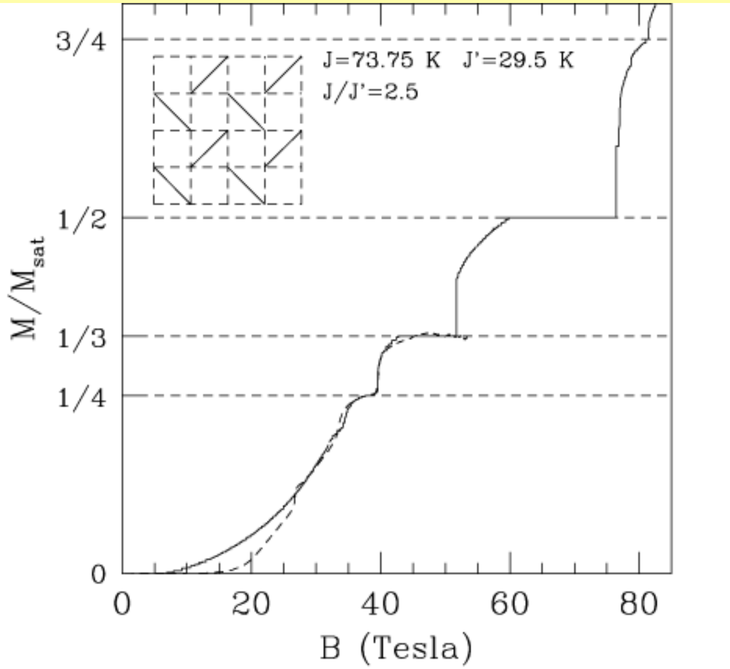


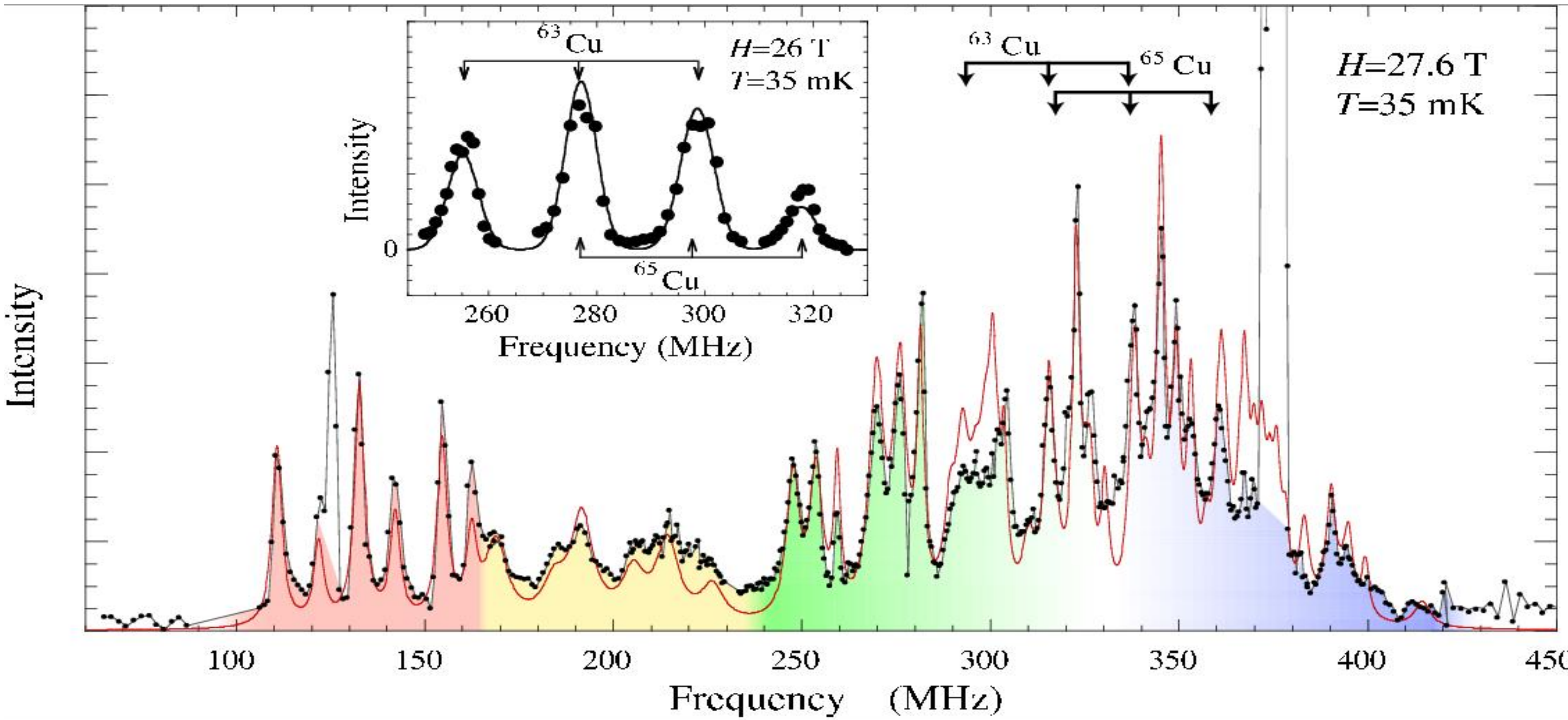
FIG. 1. Comparison between the magnetization curve of measured by Onizuka *et al.* (dashed line) and the mean-field result (solid line). Inset: Shastry-Sutherland lattice. The exchange interaction is on black links and on the dotted ones.

- Best agreement with experimental data
- Map spins to spinless fermions via C-S theory
- **Assume uniform local magnetization**

Not consistent with expt. or intuitive picture

NMR at the 1/8 plateau

NMR experiment

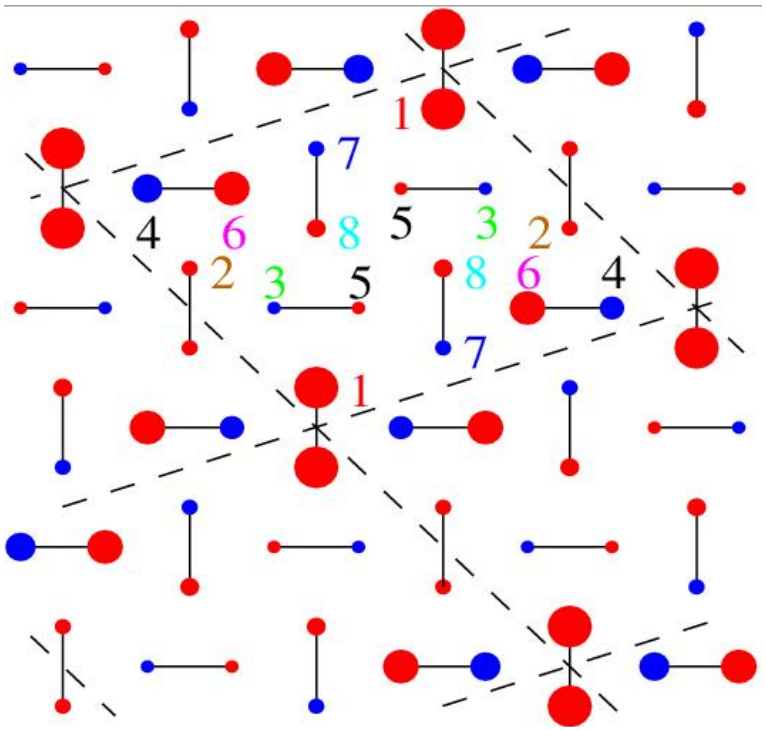


At least 11 different sites!

K. Kodama, et. al. , Science ‘02

NMR at the 1/8 plateau

Magnetization profile at 1/8

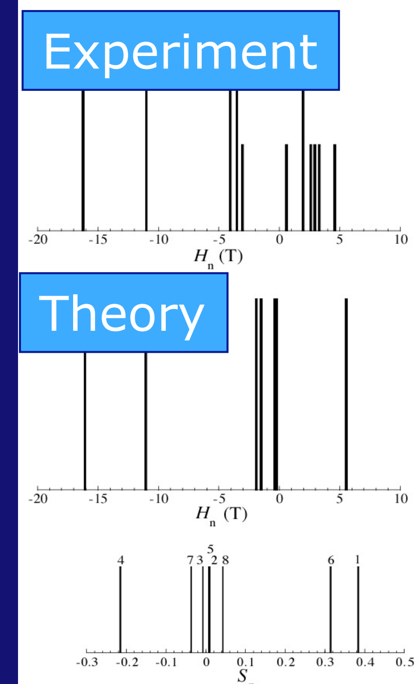


- **Magnetization opposite to field**
- **Magnetization in field direction**

Symmetry breaking

16 sites/unit cell 8-fold degenerate GS

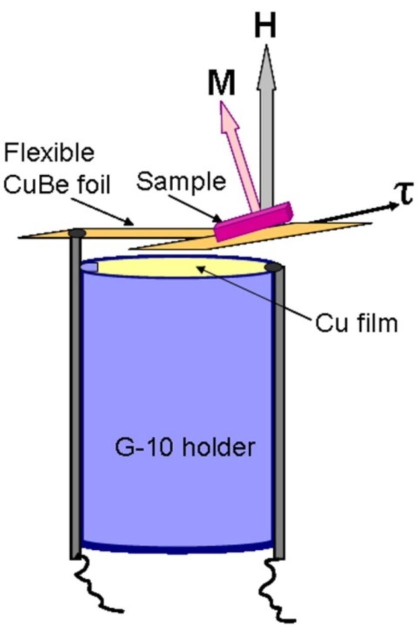
Lattice distortion: selection of one ground state



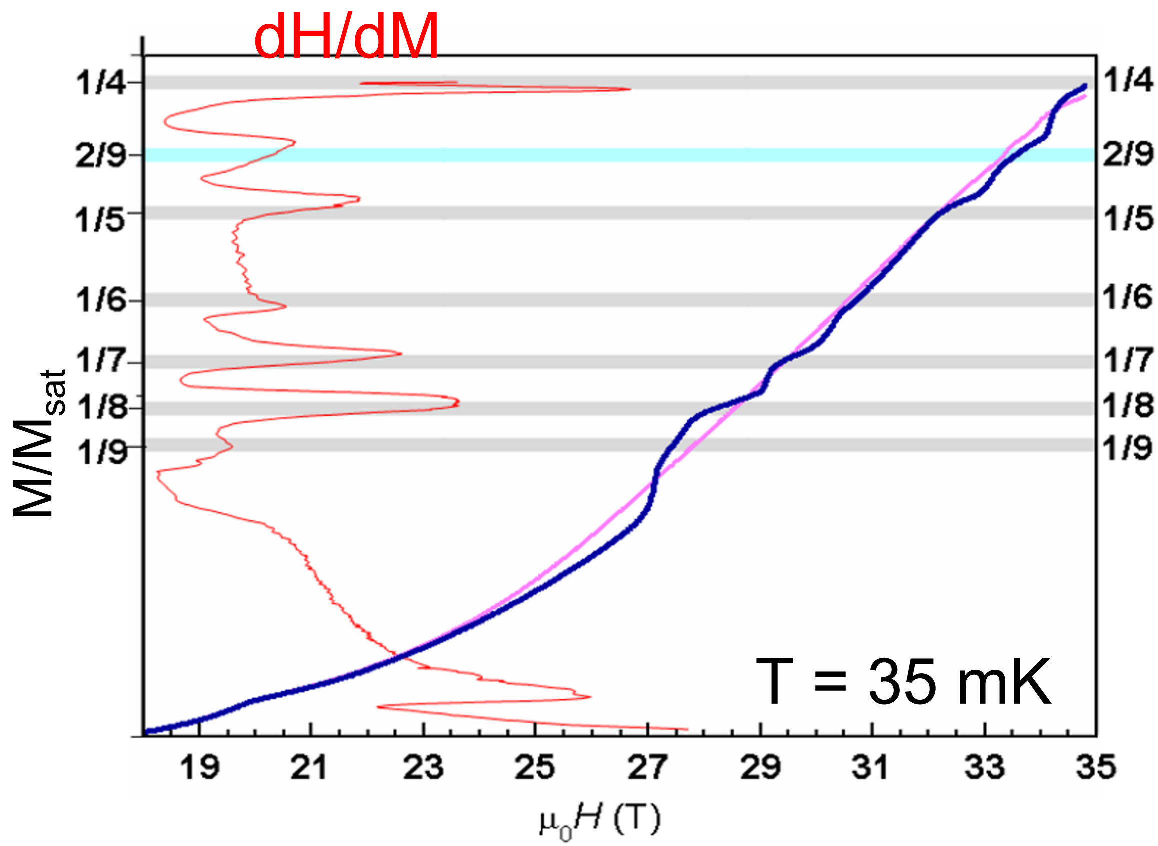
K. Kodama, et. al. , Science '02

New Measurement of M vs. H

Complete 1/n series of plateaus revealed



cantilever magnetometer (on a rotator)



- ✓ Previous measurements were at > 150 mK
- ✓ Plateaus rounded due to D-M interaction

S. Sebastian, et.al., Proc. Natl. Acad. Sci (USA) **105**, 2057 (2009)

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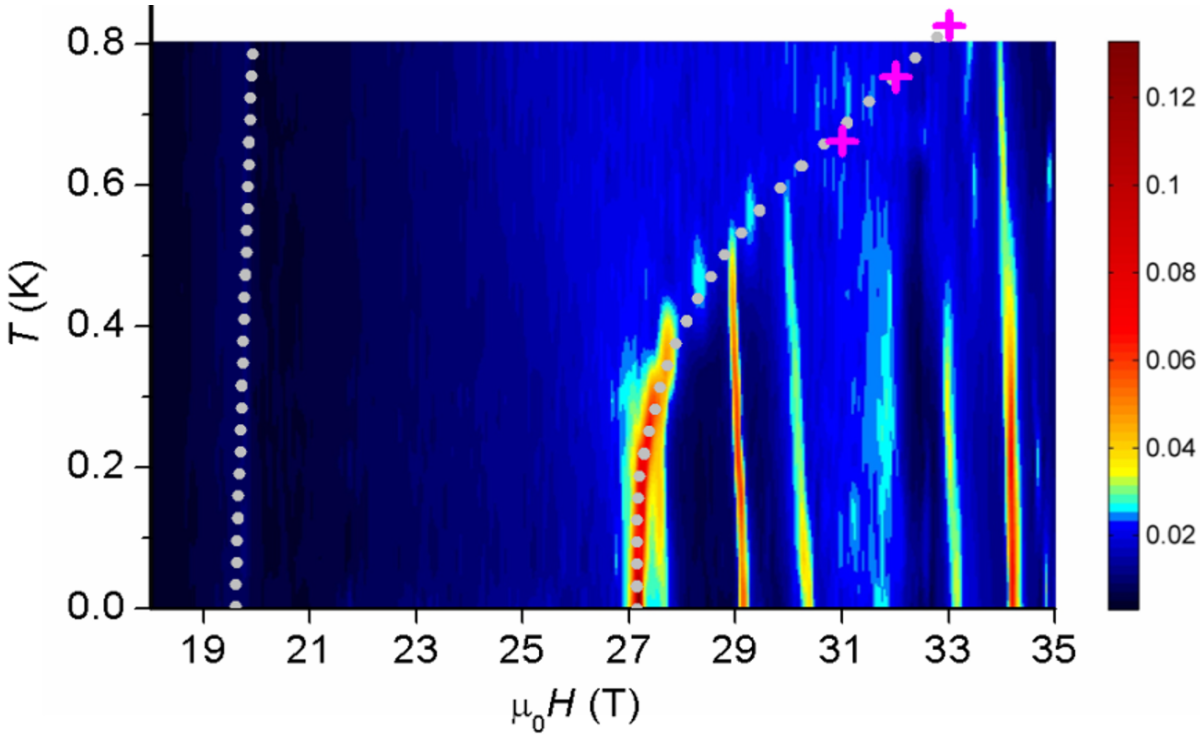
Experimental Phase Diagram



- Fine structure due to competing energy scales
- Consequence of interplay between strong interactions, frustration
- How do we identify these phases?

1/n hierarchy of magnetization plateaus

<u>Theoretical challenge: explain the emergence of the plateaus from a microscopic model</u>

Start with the C-S theory and improve on it

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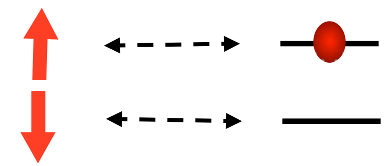
Chern-Simons theory: S=1/2 to spinless fermions

$$H = \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S_j} - B \sum_i S_i^z$$



Matsubara-Matsuda

S=1/2 to hard-core bosons

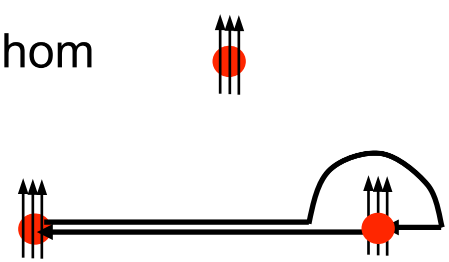
$$H = \sum_{ij} \frac{J_{ij}}{2} (b_i^\dagger b_j + b_j^\dagger b_i) + J(n_i - 1/2)(n_j - 1/2) - B \sum_{\mathbf{i}} n_{\mathbf{i}}$$

Jordan-Wigner

hard-core bosons to spin-less fermions
with attached flux

$$H = \sum_{i,j} \frac{J_{ij}}{2} (f_i^\dagger e^{iA_\mu(i)} f_j + f_j^\dagger e^{-iA_\mu(i)} f_j) + J_{ij}(n_i - \frac{1}{2})(n_j - \frac{1}{2}) - B \sum_i n_i$$

Aharonov-Bhom effect



The bosonic statistics is recovered if a flux quantum is attached to each fermion.

G. Misguich, T. Jolicoeur and S. M. Girvin, Phys. Rev. Lett. **87,** 097203 (2001).

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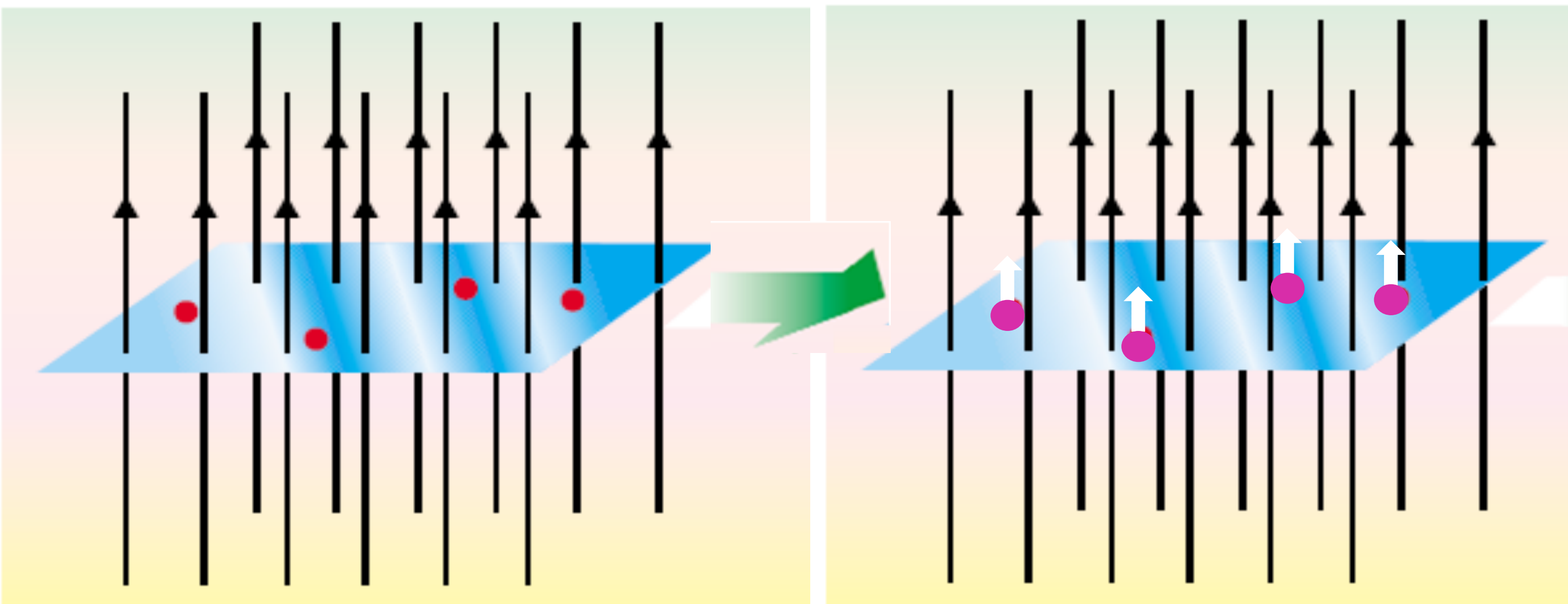
Mean Field: the flux is uniformly distributed over the whole lattice

Statistical flux Φ smeared out and treated as a mean field (Chern Simons treatment).

$$\phi = 2\pi\rho$$



Applied magnetic field = chemical potential ⇨ controls total number of fermions ⇨ determines the total statistical flux

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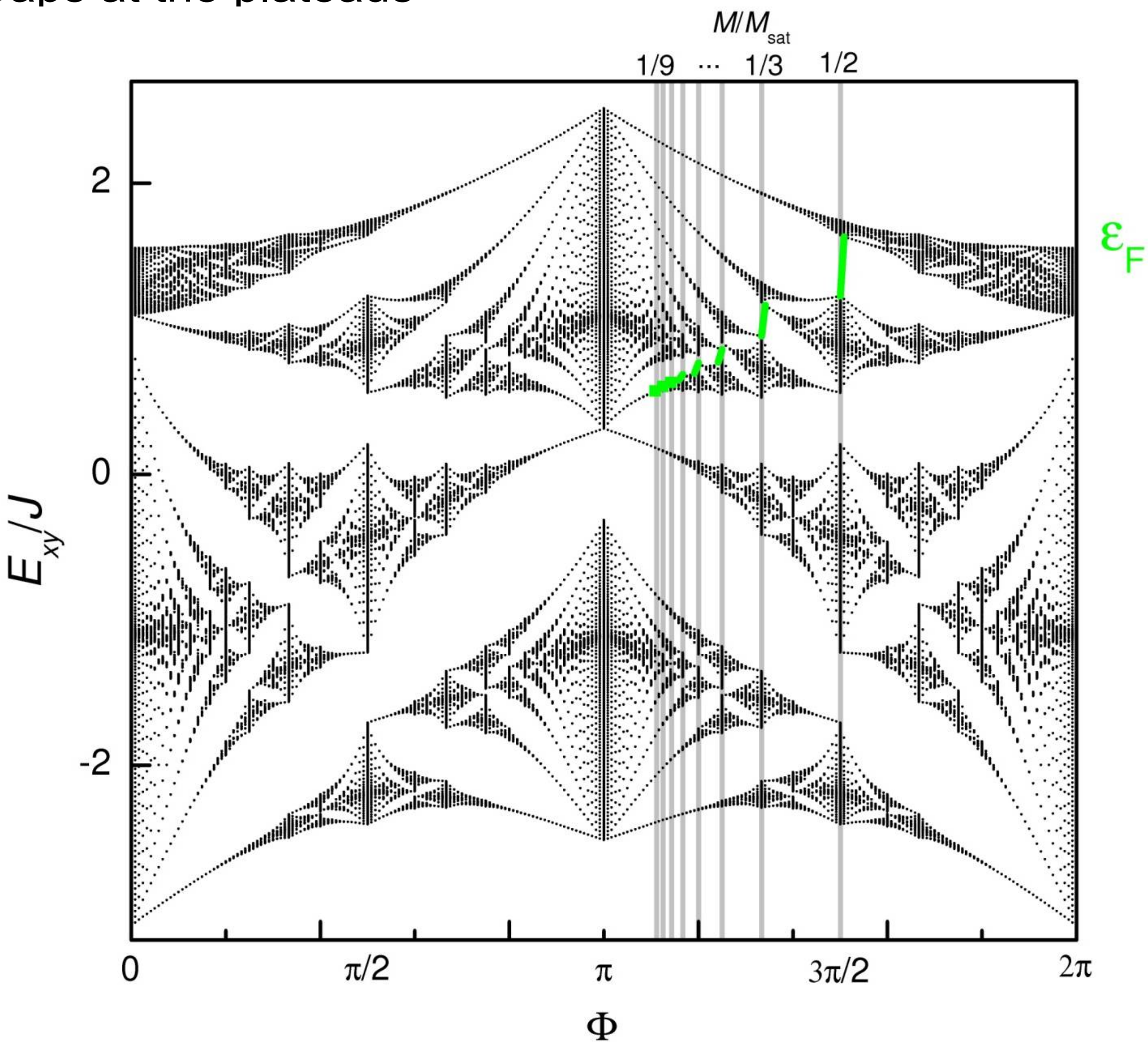
Hofstadter Butterfly

- Energy levels of the non-interacting part of the Hamiltonian
- The total flux is determined by the net magnetization (fermion density)
- Gaps at the plateaus



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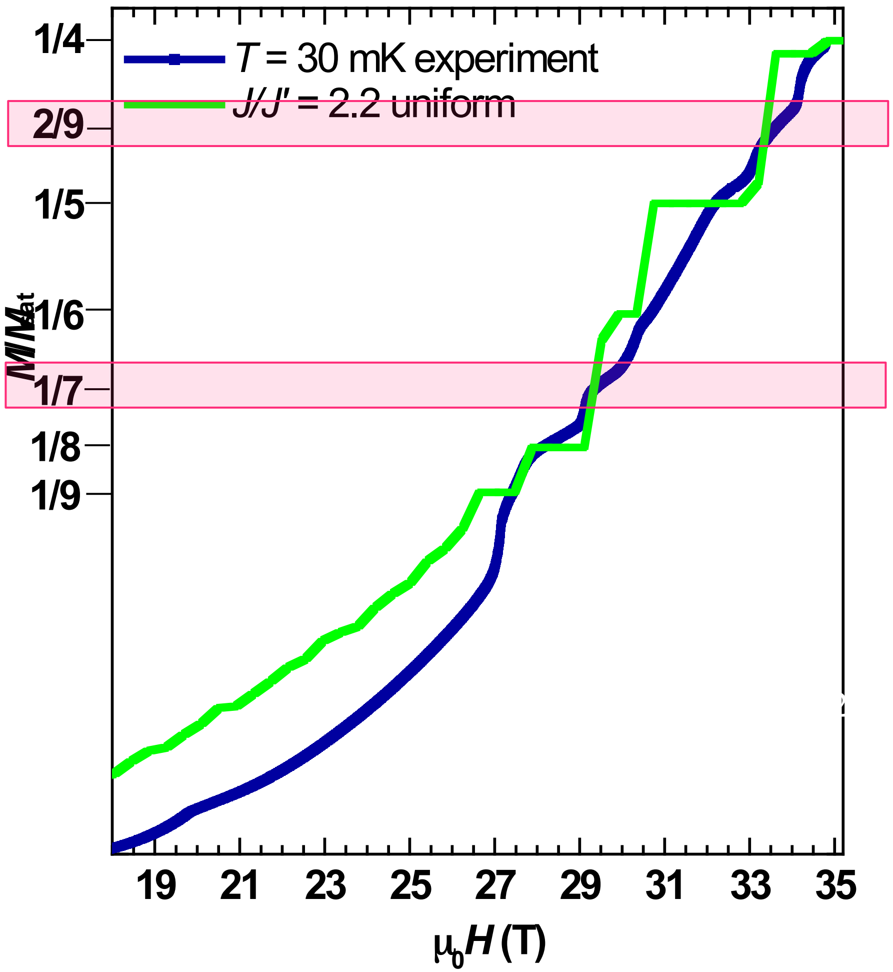
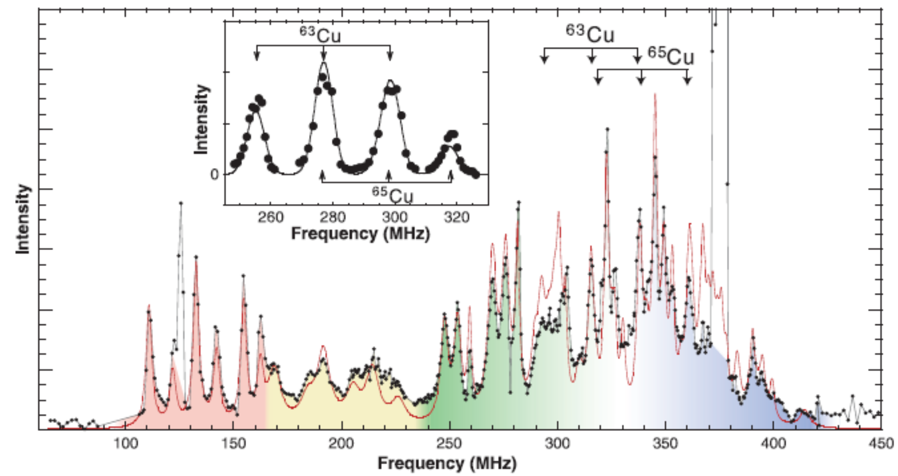
Uniform Incompressible Solutions: Incomplete Picture

Uniform Mean Field: $J_{ij} n_i n_j \rightarrow J_{i,j} \rho^2$



Optimal ratio: J'/J = 2.2



Need to consider density-density interactions: translational symmetry-breaking

K. Kodama et al. *Science* ***298* 395 (2002)**

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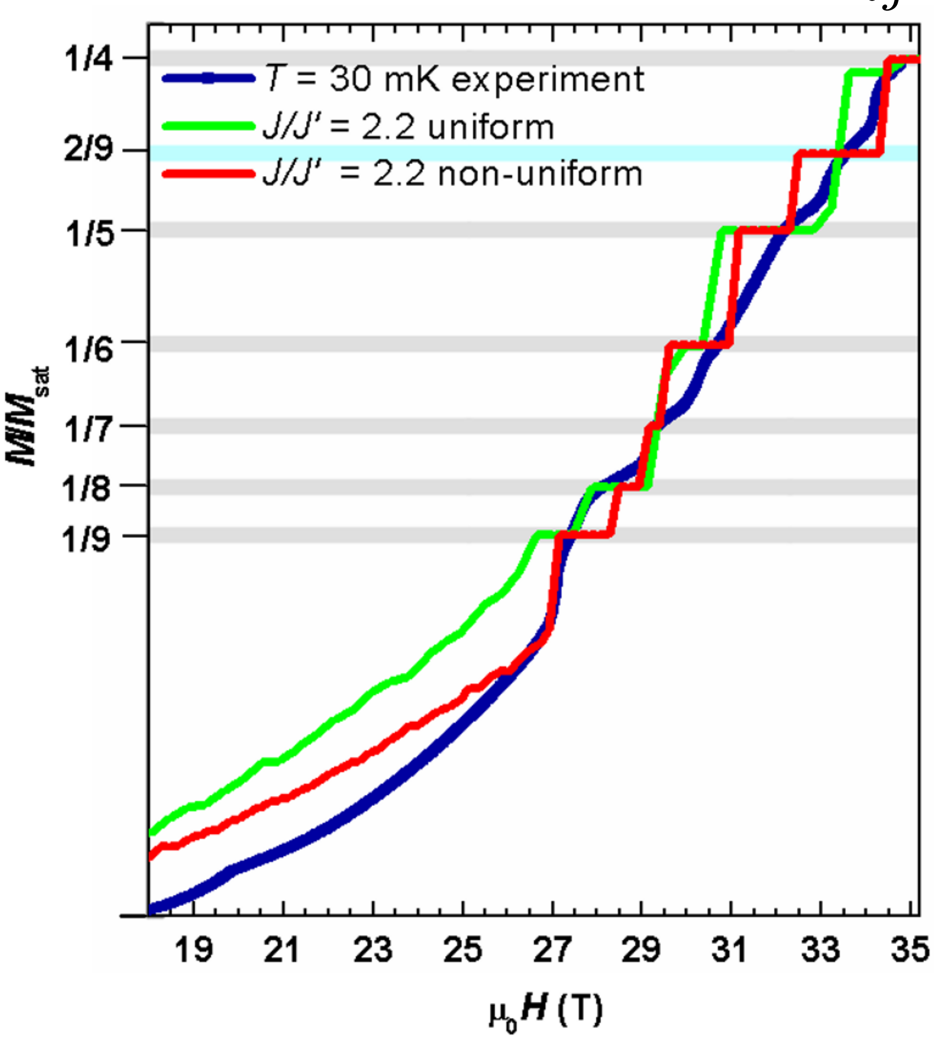
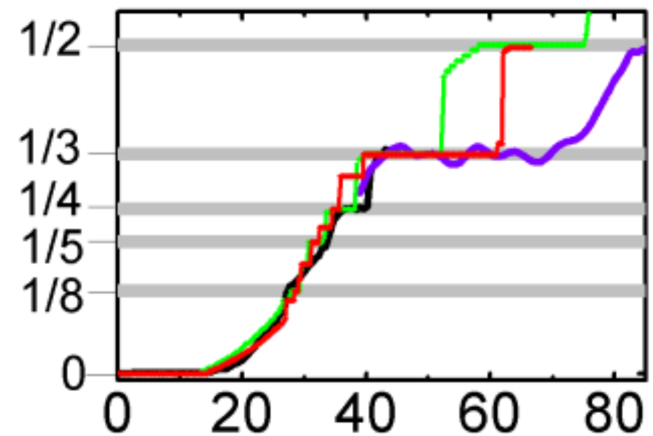
Chern-Simons Theories: M vs. H

Non-Uniform Mean Field: $J_{ij} n_i n_j \rightarrow J_{i,j}(\rho n_i + \rho n_j - \rho^2)$





- **Chern-Simons treatment with relaxed local densities**
- **Entire sequence of plateaus captured**

S. Sebastian, et.al., Proc. Natl. Acad. Sci (USA) **105**, 20157 (2009)

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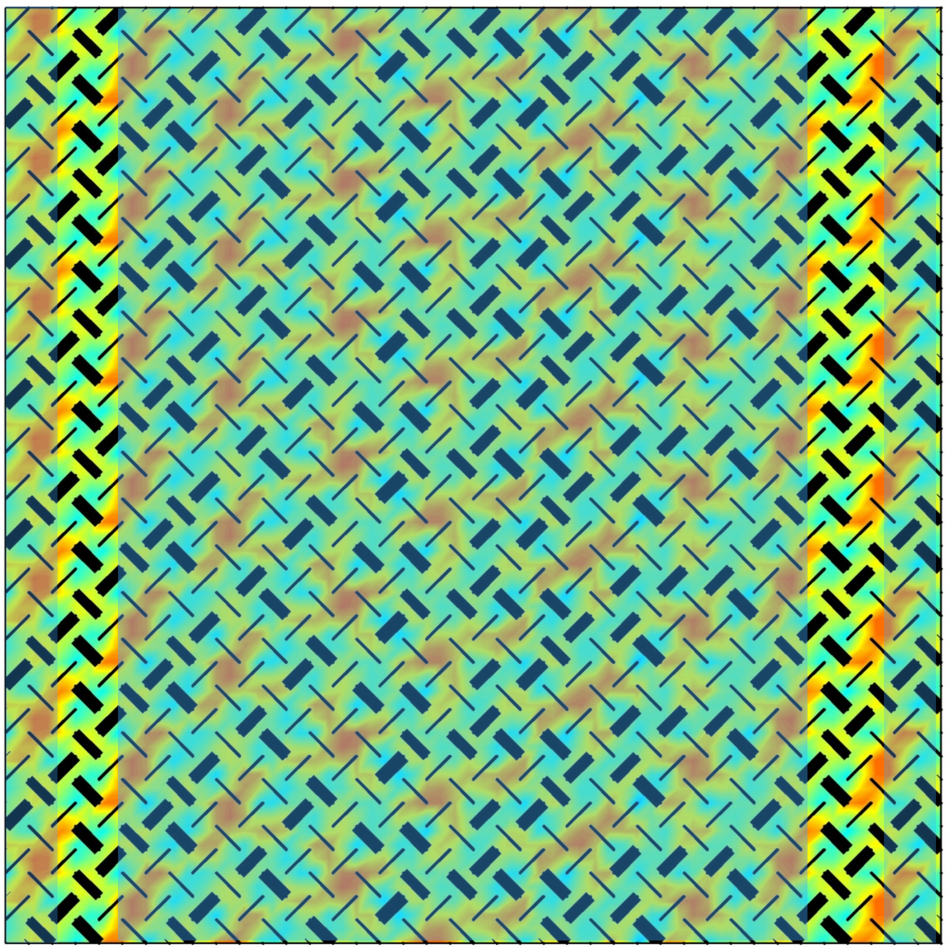
Crystal obtained for the 1/8 plateau

Stripe superstructures modulate underlying short-range structures



- Example density profile for $m_z/m_{sat} = 1/8$
- Stripe superstructures
- Underlying clusters of polarized dimers

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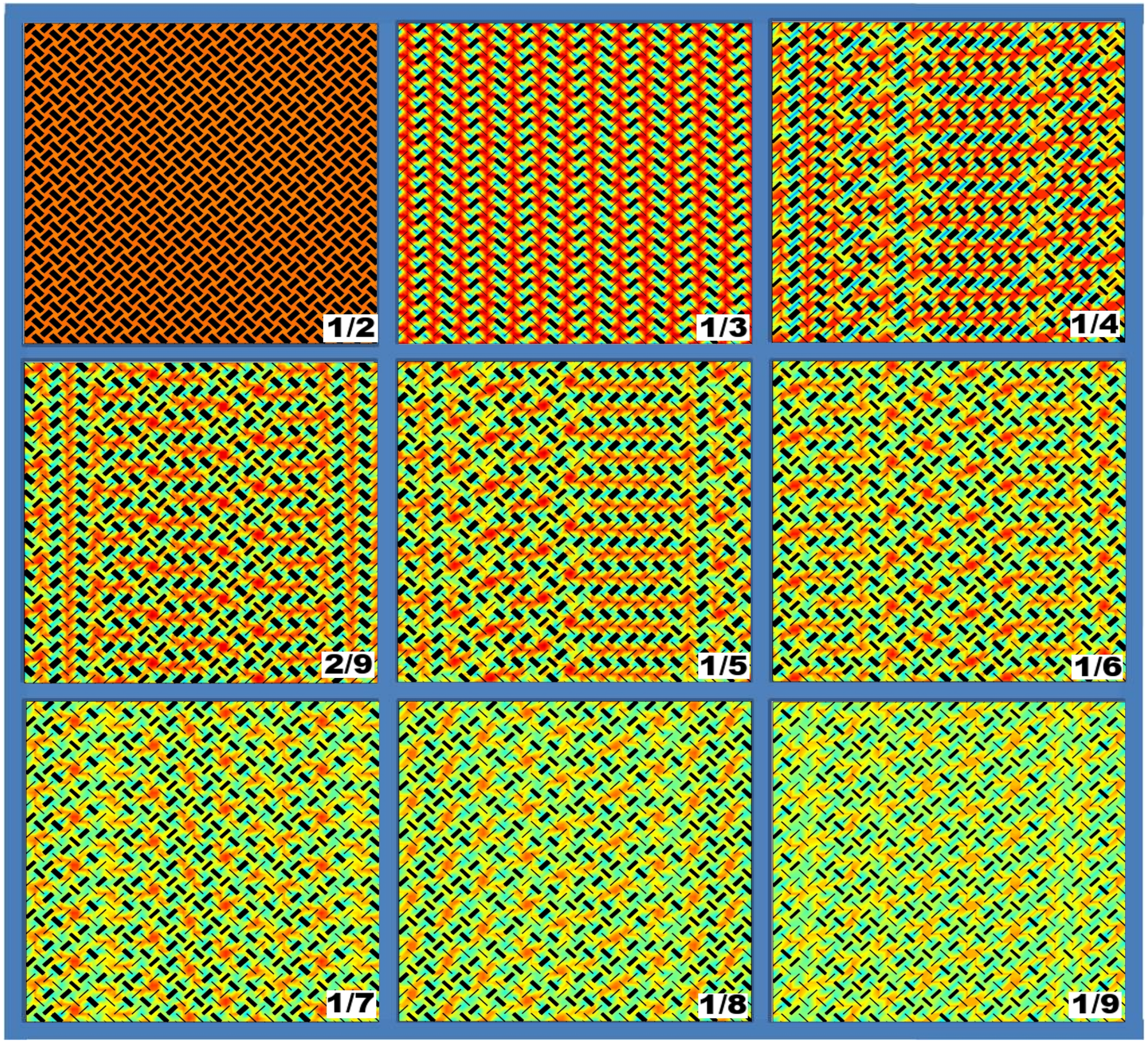
Crystals obtained for each plateau

Stripe superstructures modulate underlying short-range structures



Other realizations of the SSL model

- Rare earth tetraborides and
- R2T2M

TbB_4	xy	40^0K
DyB_4	$Quadrupolar\ \ order$	15^0K
TmB_4	$Isinglike$	11.7^0K
HoB_4	$Isinglike$	7^0K
ErB_4	$Isinglike$	15^0K
Yb_2Pt_2Pb	$mixed(FM + AFM)$	2^0K

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Rare earth tetraborides

Generic features

- Low saturation field (~ 10T) – low-T neutron scattering possible
- Span wide parameter regime including magnetic ground state at zero field
- Higher spins, single ion anisotropies ⟹ effective exchange anisotropy
- Additional interactions

Extensive insight into the interplay between competing strong interactions and geometric frustration in the Shastry-Sutherland lattice

Metallic ground state – interaction of itinerant electrons with localized magnetic moments in a frustrated configuration – interesting magneto-electric effects

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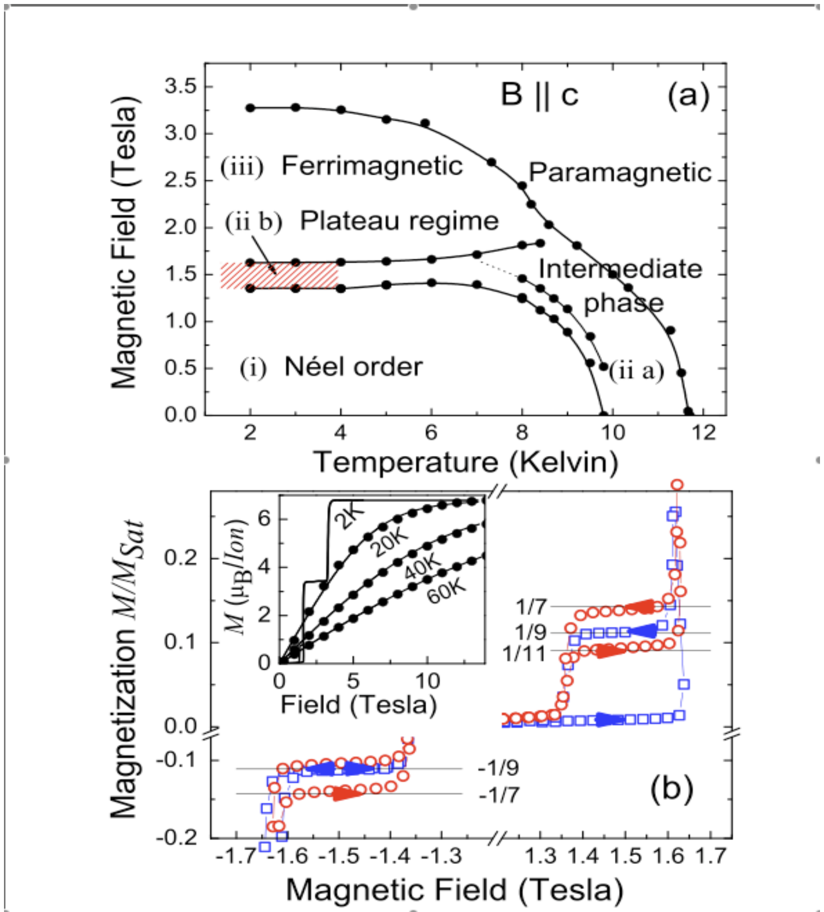
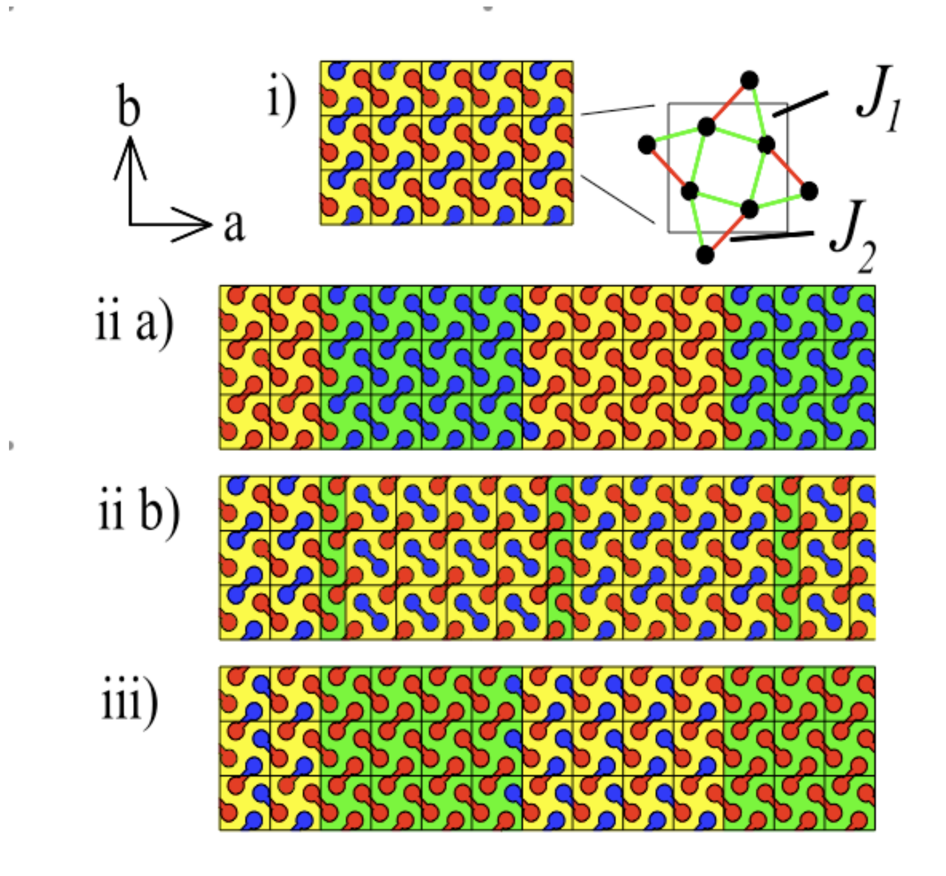
Magnetization Plateaus in TmB_4

Stripe superstructures modulate underlying short-range structures

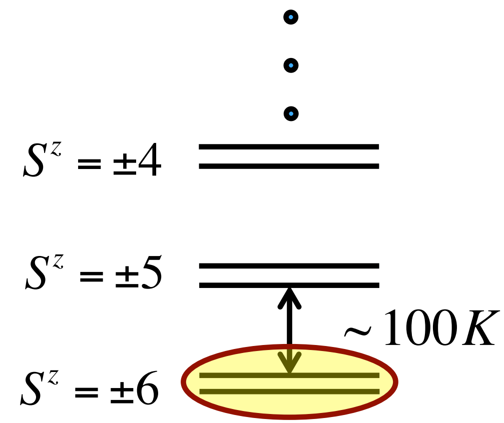




K. Siemensmeyer et. al., PRL <u>**101**</u>, 177201 (2008)

Low energy effective model for TmB_4

- Large magnetic moment for Tm^{3+}: $S = 6$
- Large single-ion anisotropy: $-D\left(S_i^z\right)^2, D/J \approx 50$



Effective low energy model involving the lowest 2 levels

$$H = \sum_{\langle i,j \rangle} J_{ij}^z S_i^z S_j^z + J_{ij}^{xy}\left(S_i^x S_j^x + S_i^y S_j^y\right)$$

$$J_{ij}^{xy} < 0 \qquad J_{ij}^z \gg \left|J_{ij}^{xy}\right|$$ Ising limit

⇓

Ferromagnetic exchange term – sign problem in QMC simulations alleviated

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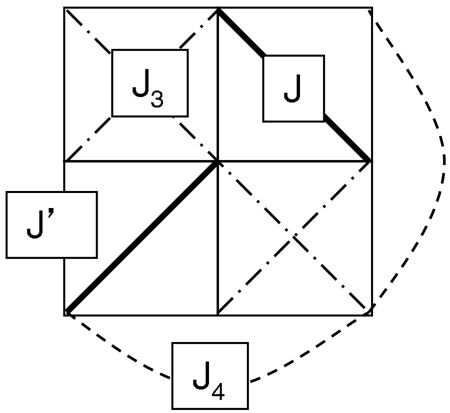
Low energy effective model for TmB_4

Start with the Shastry Sutherland model in the Ising limit ⇨ inconsistency with experimental observation



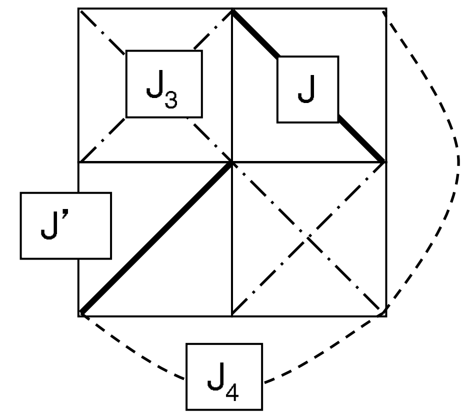
Need additional interactions

Ferromagnetic 3rd neighbor interaction necessary to explain plateau structure observed in TmB_4.

PRB **80**, 180405 (2009); PRB **82**, 214404 (2010)

Generalized Shastry-Sutherland model

- S=1/2 XXZ model with large Ising anisotropy
- J and J' along the SSL lattice axes
- Additional 3rd neighbor interaction (FM or AFM)
- NNN interaction along the diagonals of the plaquettes with no J
- Same anisotropy for all interactions.



- Rich variety of magnetic phases
- Potentially realized in the different members of the rare-earth tetraborides

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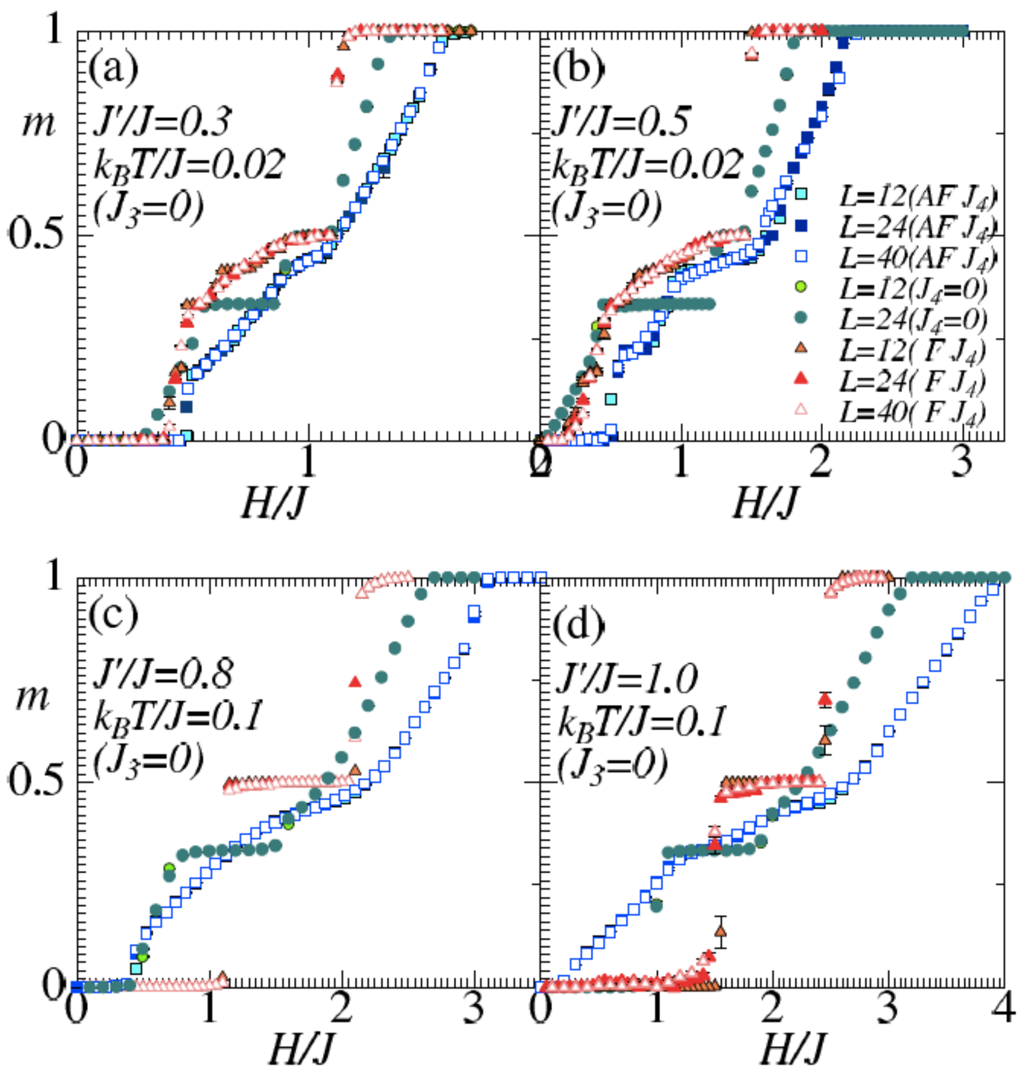
Magnetization plateaus in the generalized SSL model



Different sequence of plateaus as the parameters are varied

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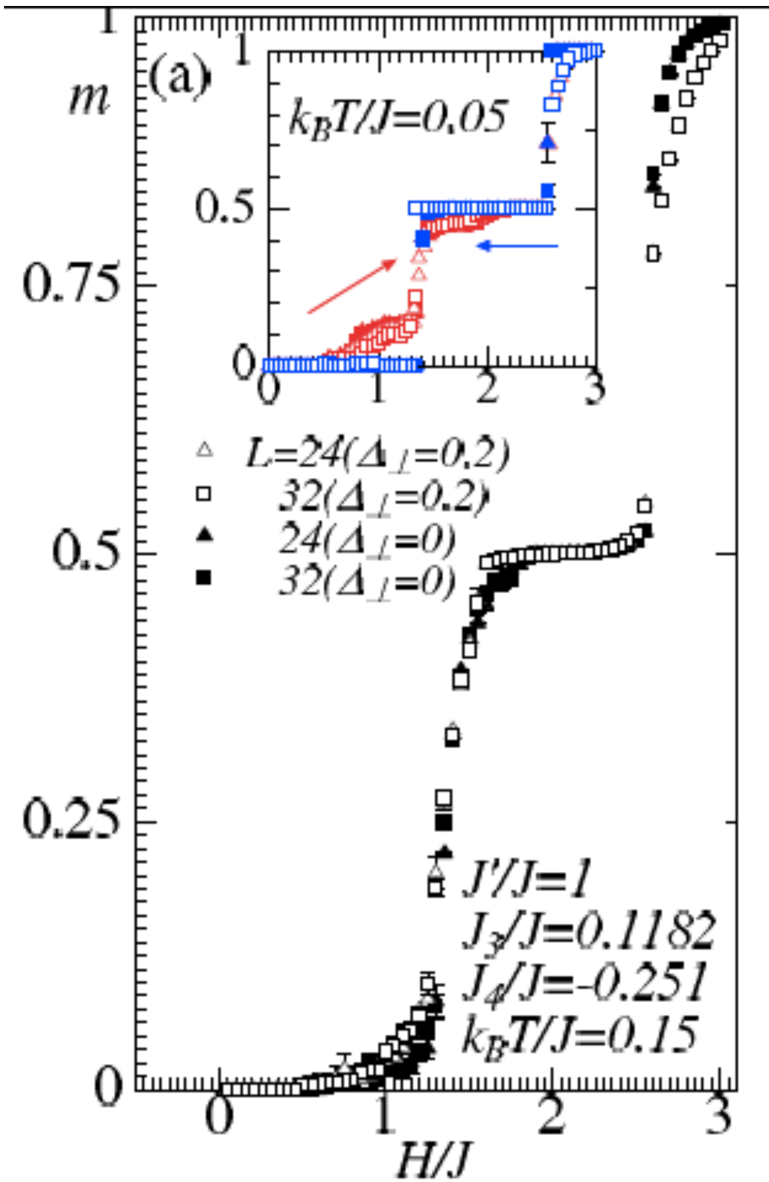
Modeling TmB_4



- ✓ Correct critical values for the $m/m_s=1/2$ reproduced
- ✓ No evidence of lower magnetization plateaus for the current model
- ✓ No evidence for "hysteresis" effects.

Conclusions

- Interplay between geometric frustration, strong interaction and high magnetic field results in many novel quantum phases.
- One example is the novel sequence of 1/n plateaus discovered in $SrCu_2(BO_3)_2$.
- The emergence of the plateaus are qualitatively reproduced by the Chern Simons theory.
- The theory predicts a given sequence of crystals associated with each plateau. This can be tested with NMR and INS.
- New family of Shastry-Sutherland compounds hold great promise.

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