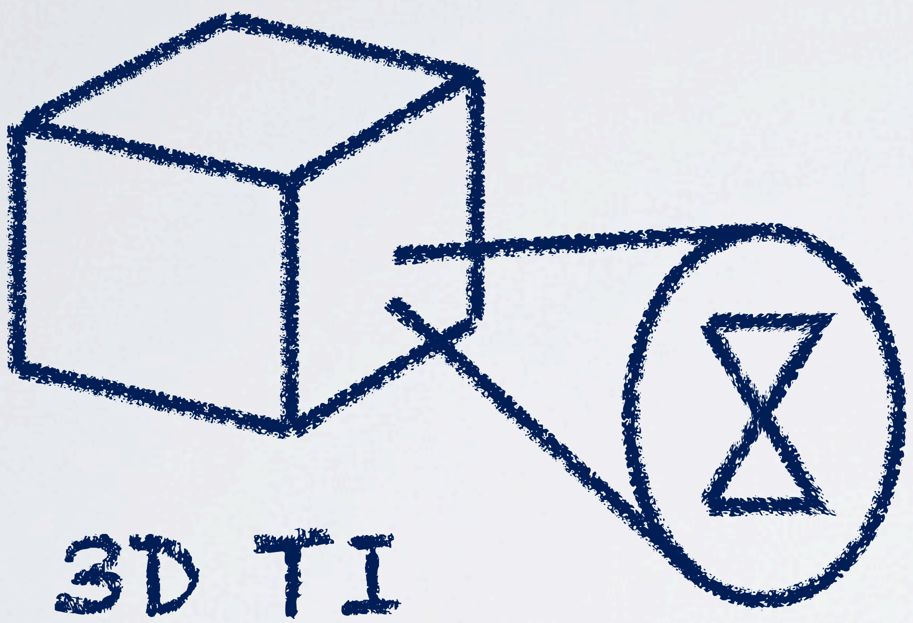
Tunable Casimir Repulsion with 3D Topological Insulators



3D TI

+

Casimir effect

Adolfo G. Grushin

NGSCES Santiago 2011

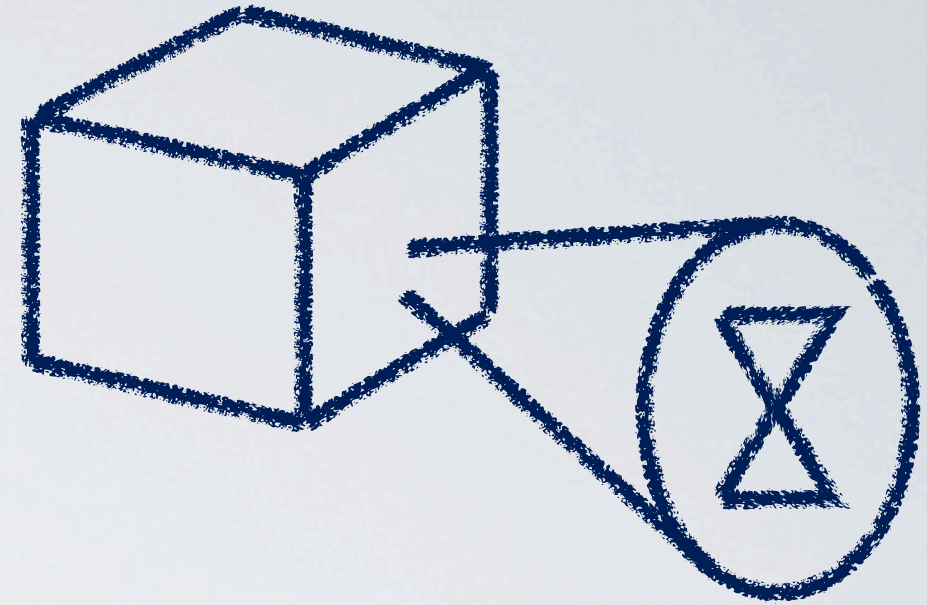
A. G. G., A. Cortijo, Phys. Rev. Lett. 106, 020403 (2011)

A. G. G., P. Rodriguez-Lopez, A. Cortijo, arXiv: 1102. 0455 Phys. Rev. B in press

http://focus.aps.org/story/v27/st1

OUTLINE

- Introduction
- Optical properties of TI's
- Repulsion! but where?
- A comment for real materials
- Conclusions



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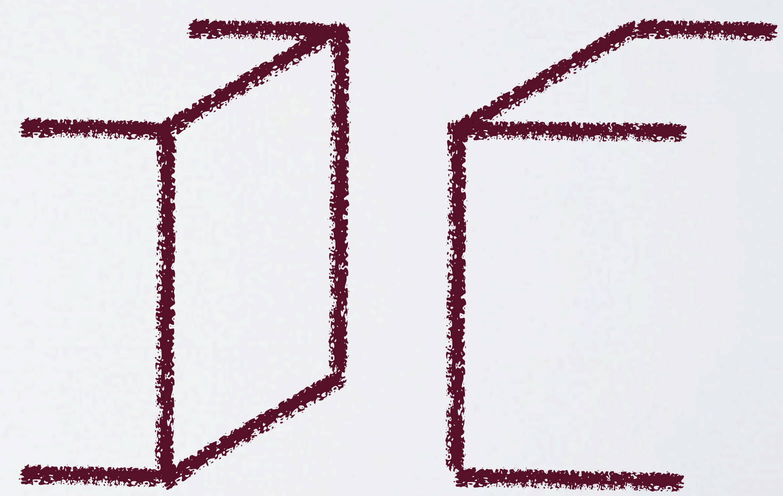

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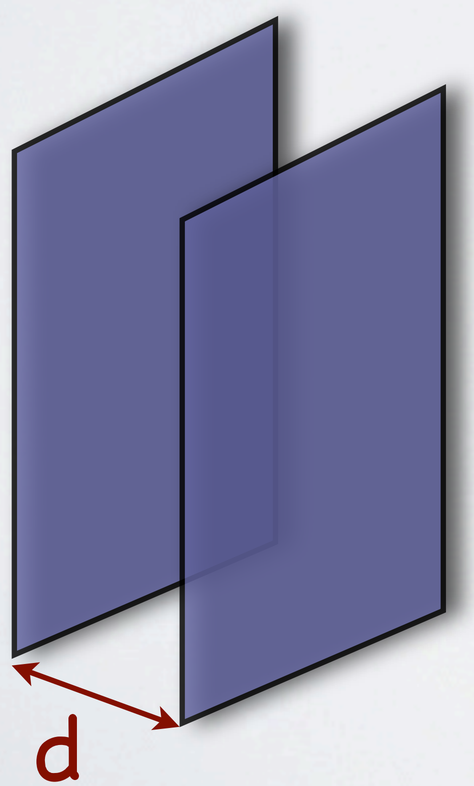

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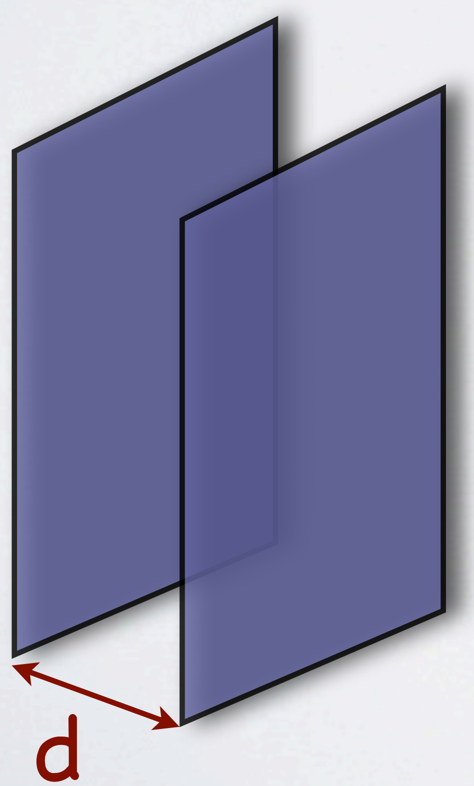

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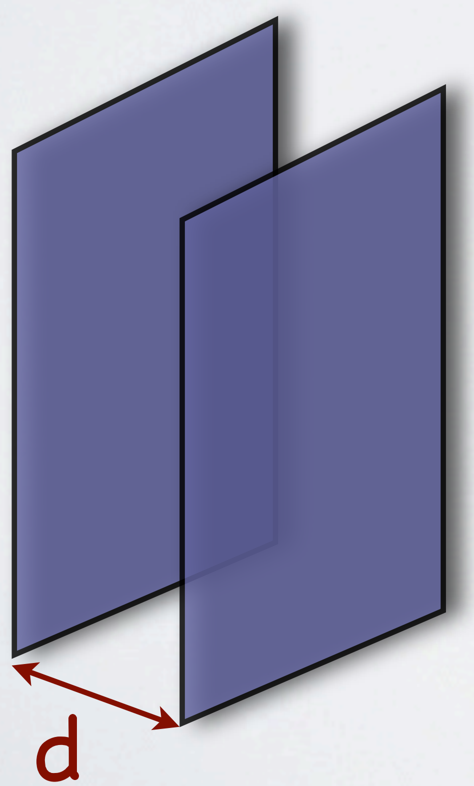

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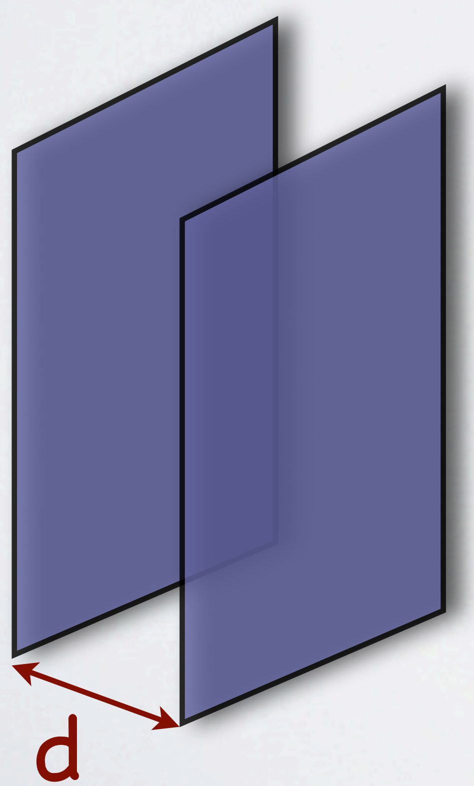

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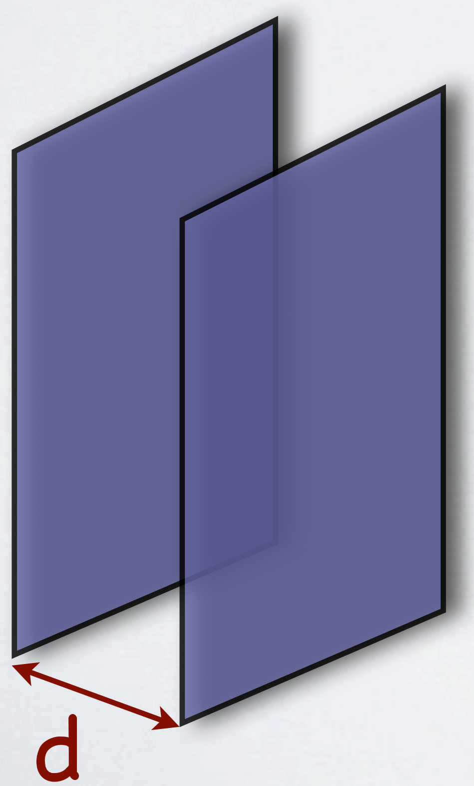

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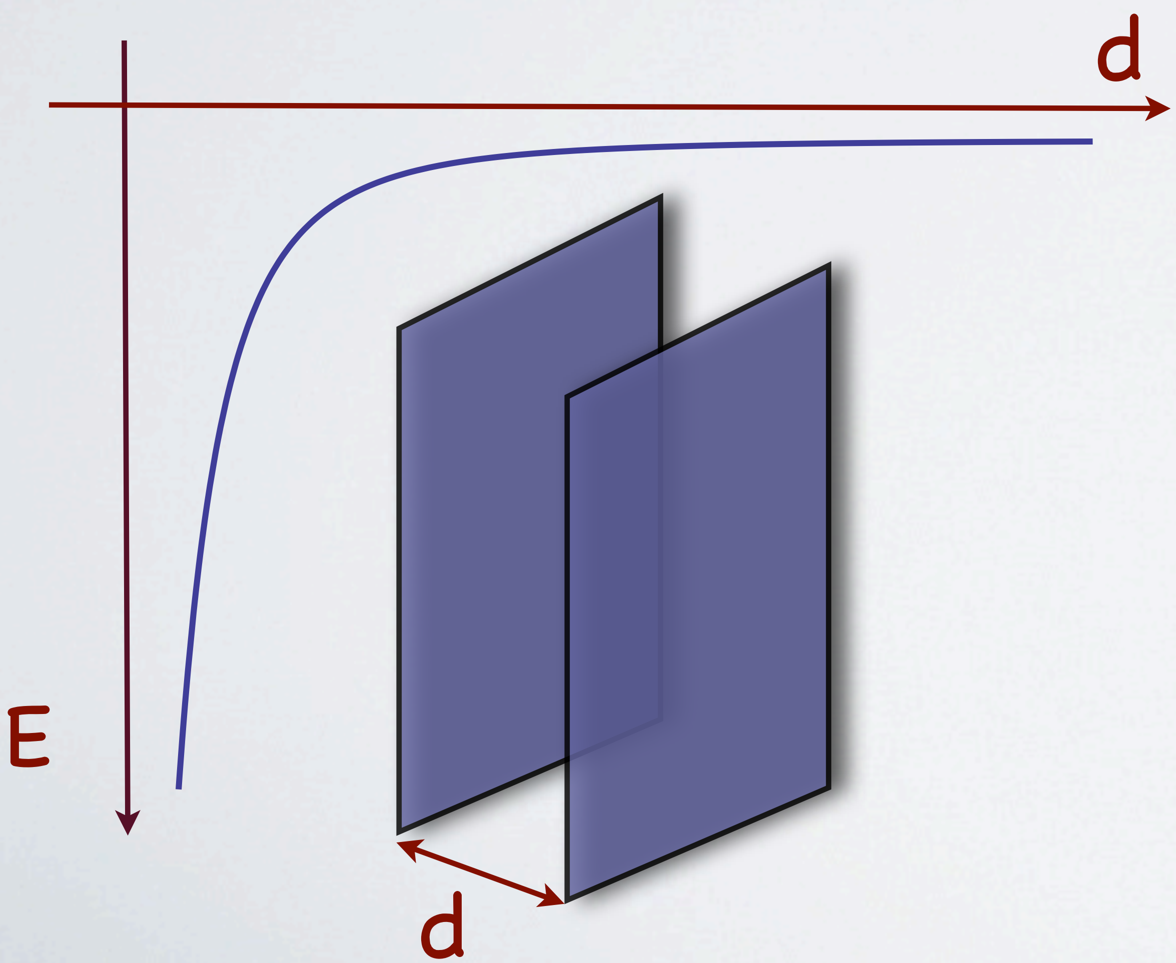

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Maritime Analogy

- The effect is analogous to the attraction of two ships in the sea

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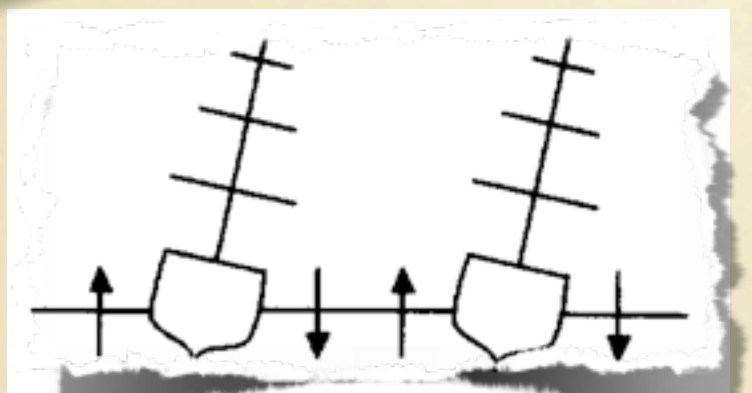
A maritime analogy of the Casimir effect

Sipko L. Boersma
Nic Beetslaan 12, 2624 XP Delft, The Netherlands

(Received 23 June 1995; accepted 16 October 1995)

At sea, on a windless day, in a strong swell, free floating ships will roll heavily. It was believed in the days of the clipper ships that under those circumstances two vessels at close distance will attract each other. Do they? The ships are harmonic oscillators in a wave field and as such analogous to two atoms in the sea of vacuum fluctuations. These atoms do attract: the van der Waals force.

Fig. 1. Two ships roll heavily on a long swell and there is no more wind to damp their rolling. In this situation a strange force, "une certaine force attractive," will pull the two ships toward each other. From P. C. Causseé: "the Mariners Album," early 19th century.



P. C. Caussé (1836) : "...une certaine force attractive."

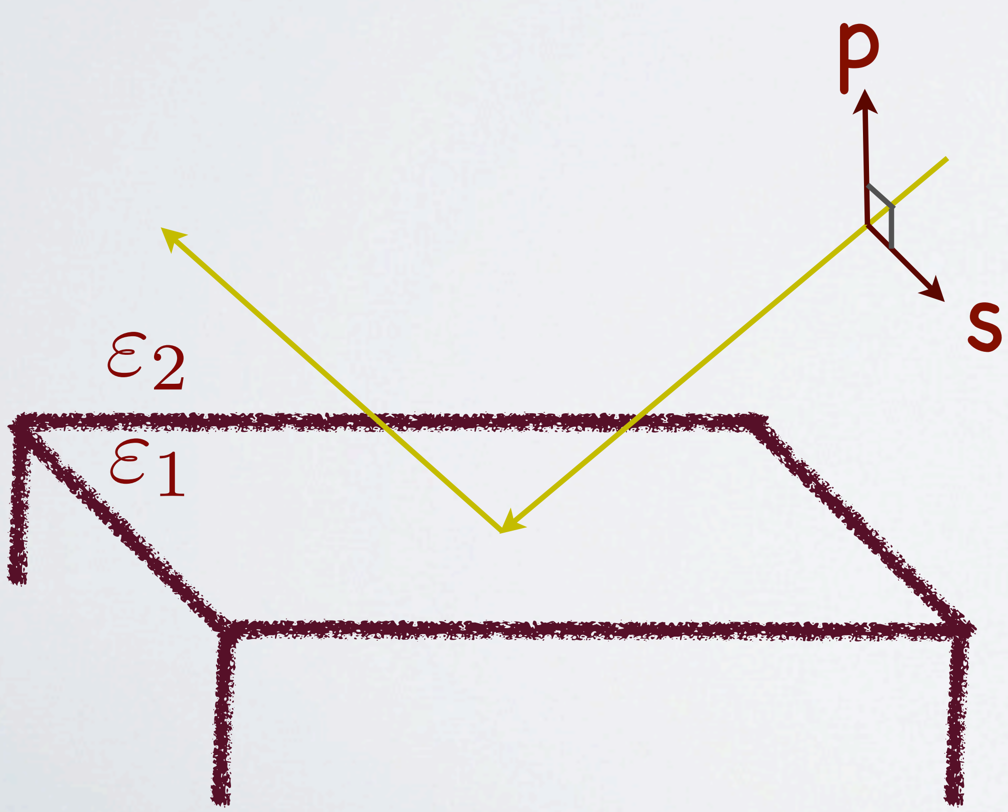
GENERALIZATION TO DIELECTRICS:

- Extension of the Casimir eff. to dielectric materials (1961):

$$\frac{E_c(d)}{A\hbar} = \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} \log\det\left[1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2k_3 d}\right]$$

I.E. Dzyaloshinskii, E.M. Lifshitz & L.P. Pitaevskii Adv. in Phys. 10, 38, (1961)

Rahi *et al.* Phys. Rev. D 80, 085021 (2009)



Reflection coefficients relate different components of the Electric field:

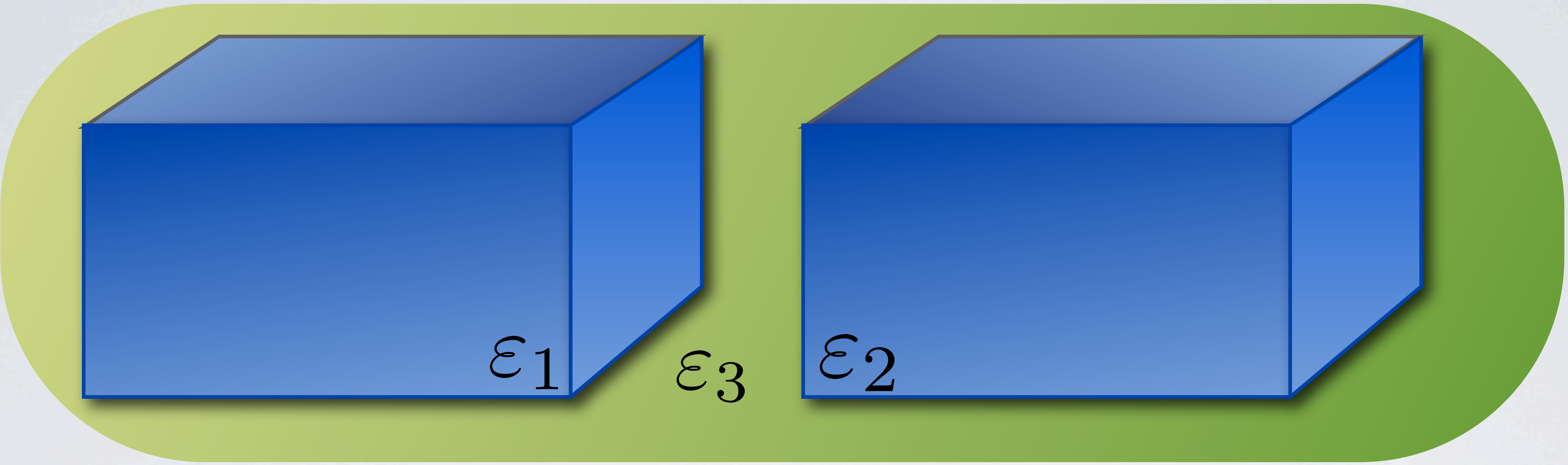
$$\mathbf{R} = \begin{bmatrix} R_{s,s}(i\xi, \mathbf{k}_\parallel) & \cancel{R_{s,p}(i\xi, \mathbf{k}_\parallel)} \\ \cancel{R_{p,s}(i\xi, \mathbf{k}_\parallel)} & R_{p,p}(i\xi, \mathbf{k}_\parallel) \end{bmatrix}$$

Dielectric:

$$\mathbf{R} = \begin{bmatrix} R_{s,s}(i\xi, \mathbf{k}_\parallel) & 0 \\ 0 & R_{p,p}(i\xi, \mathbf{k}_\parallel) \end{bmatrix}$$

THE SIGN OF THE FORCE

- Lifshitz realized that to get repulsion:



Repulsion if: $\varepsilon_1 > \varepsilon_3 > \varepsilon_2$

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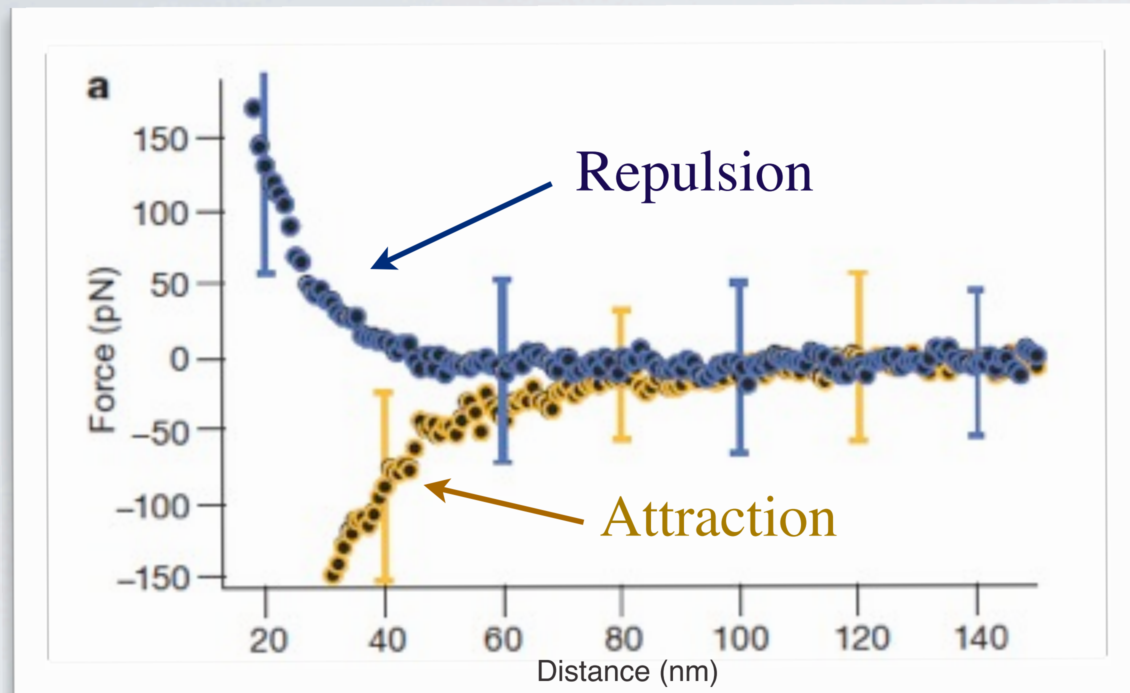
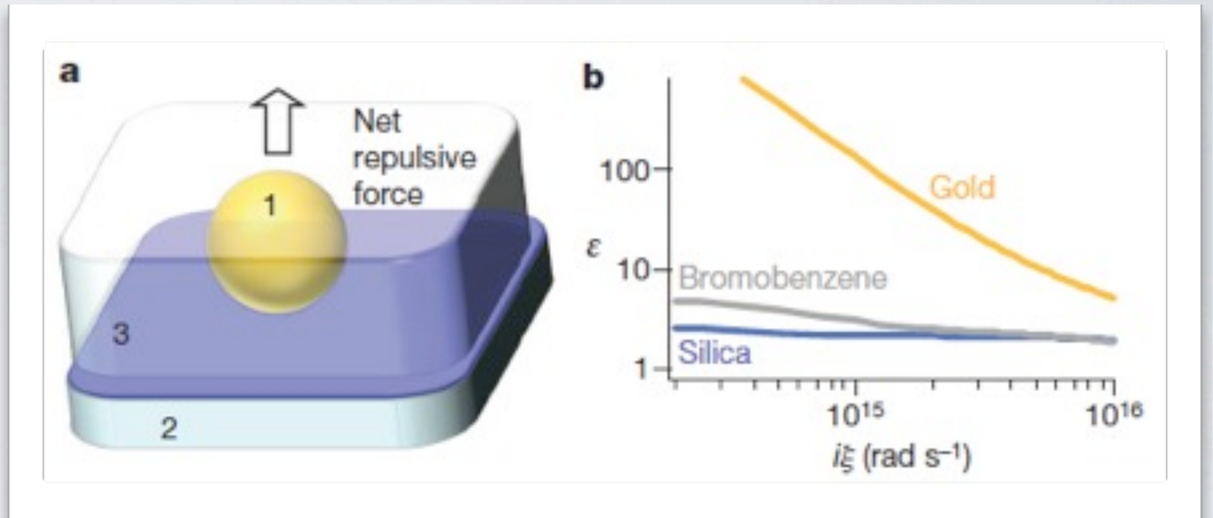


Proven experimentally!



Munday et al. Nature **457**, 170-173 (2009)

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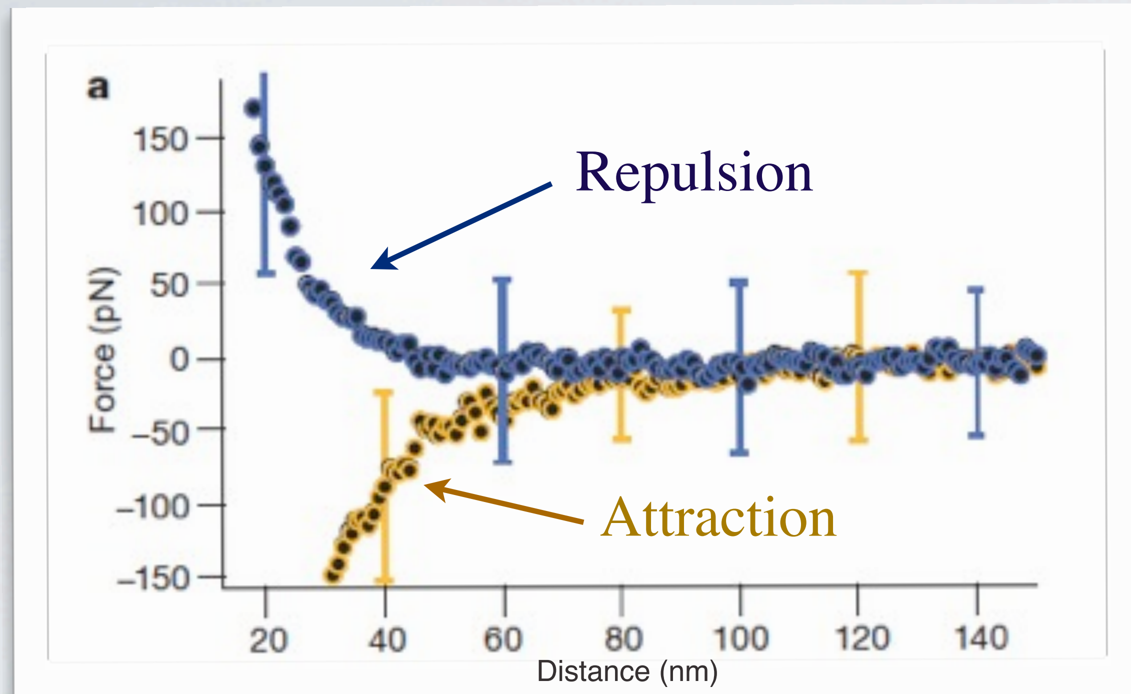

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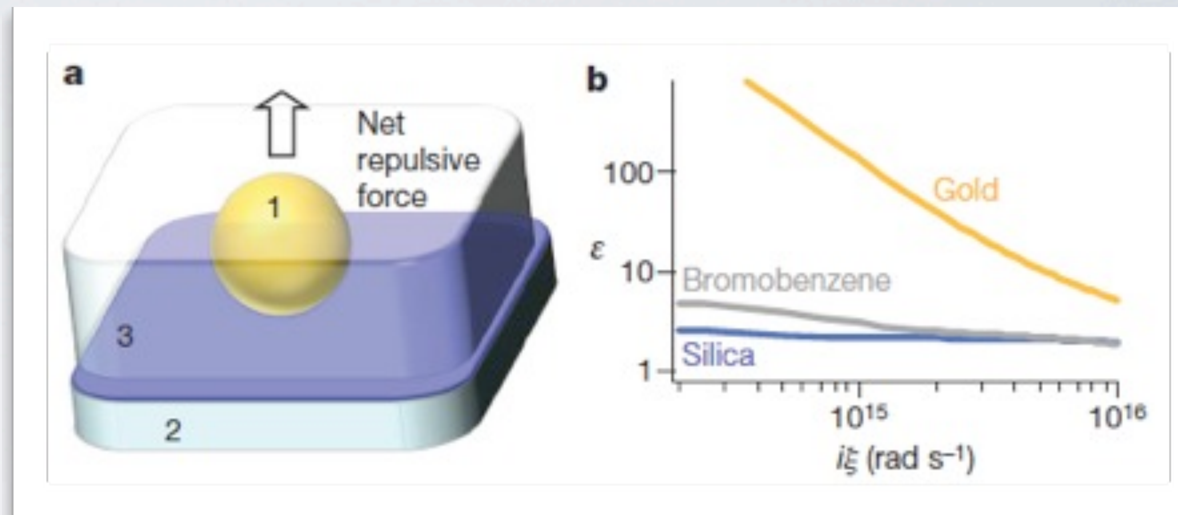

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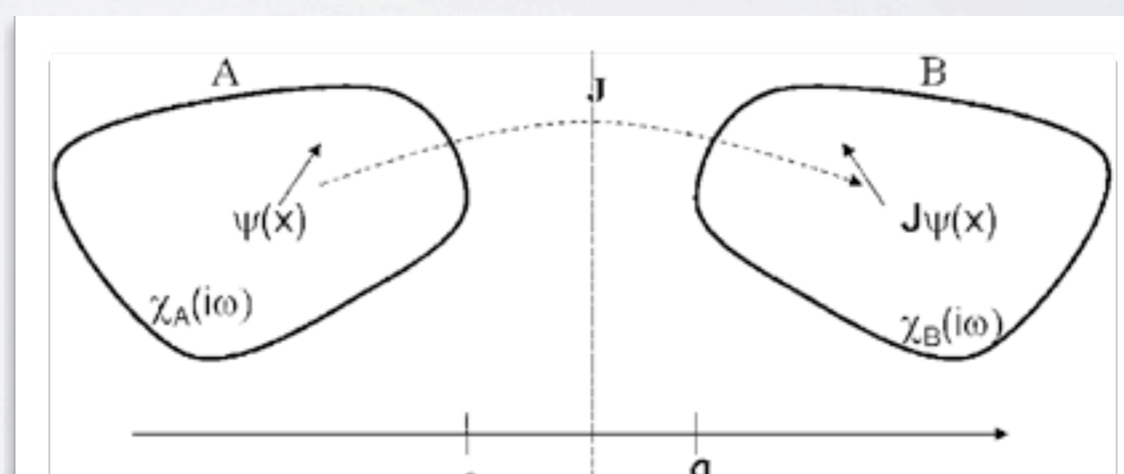
- However there are several theorems prohibiting repulsion in vacuum

Symmetric situations attract!



Kenneth, O. & Klich, I. *Phys. Rev. Lett.* **97,** 160401 (2006).

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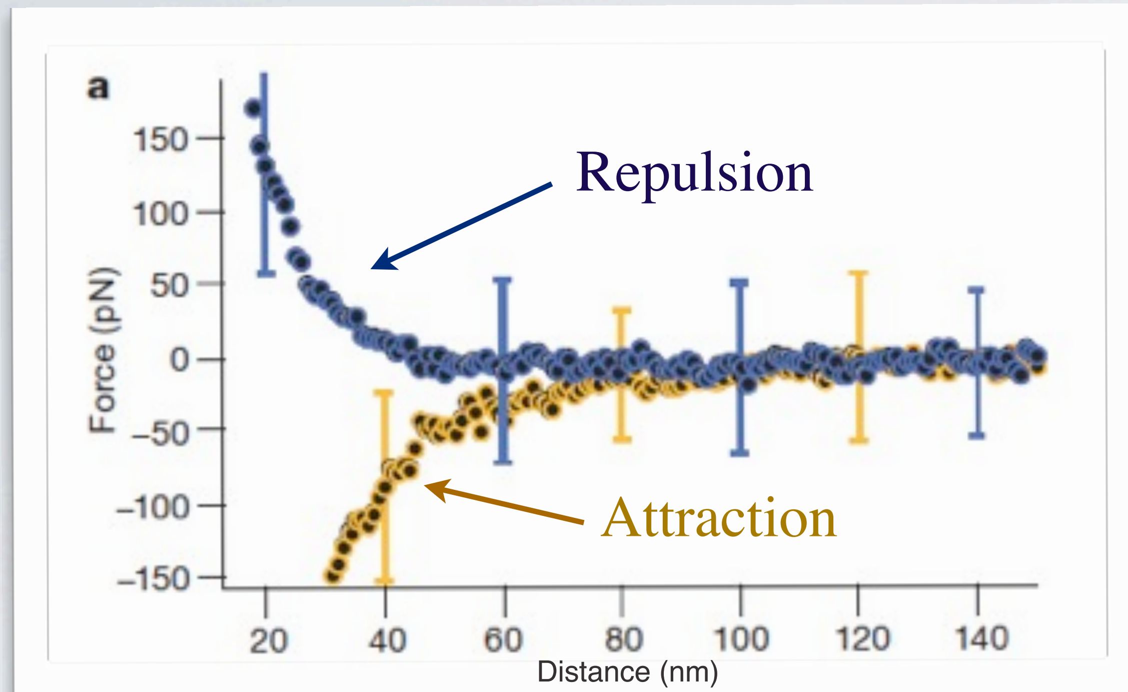

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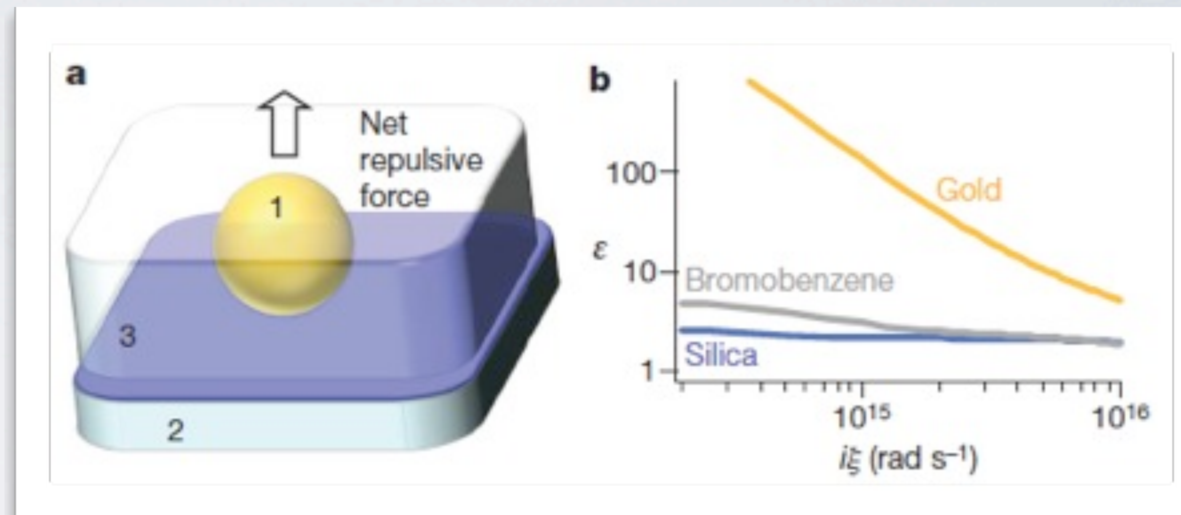

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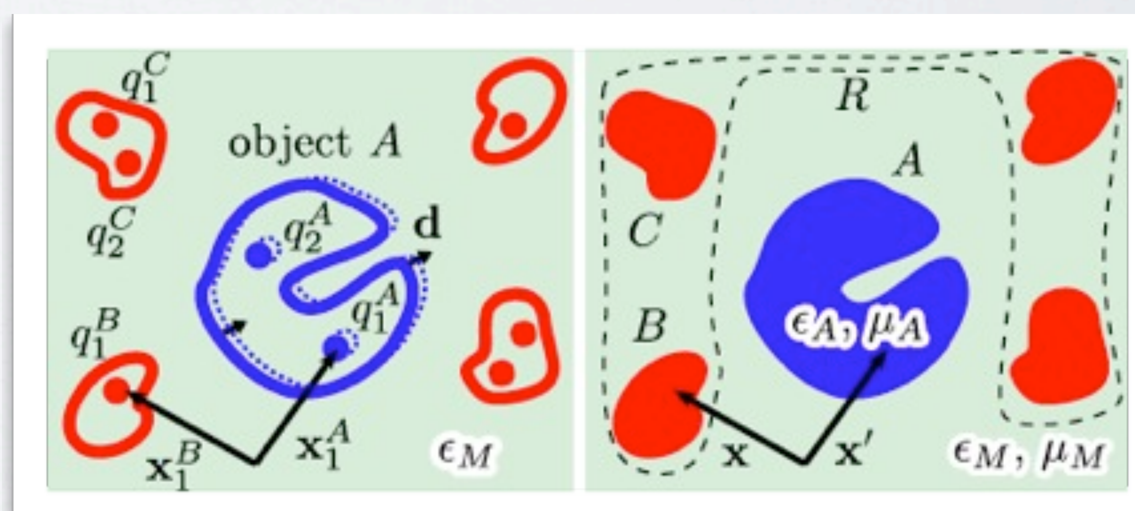
Restrictive constraints to stable equilibria:

not accessible for dielectrics in vacuum



Rahi, S. J., Kardar, M. & Emig, T. *Phys. Rev. Lett.* **105,** 070404 (2010).

WHY REPULSION?

- Fundamental question: Can we achieve a repulsive Casimir force in vacuum?

- Nanostiction and nanofriction: Reverting the Casimir force can be useful for MEMS and NEMS applications.

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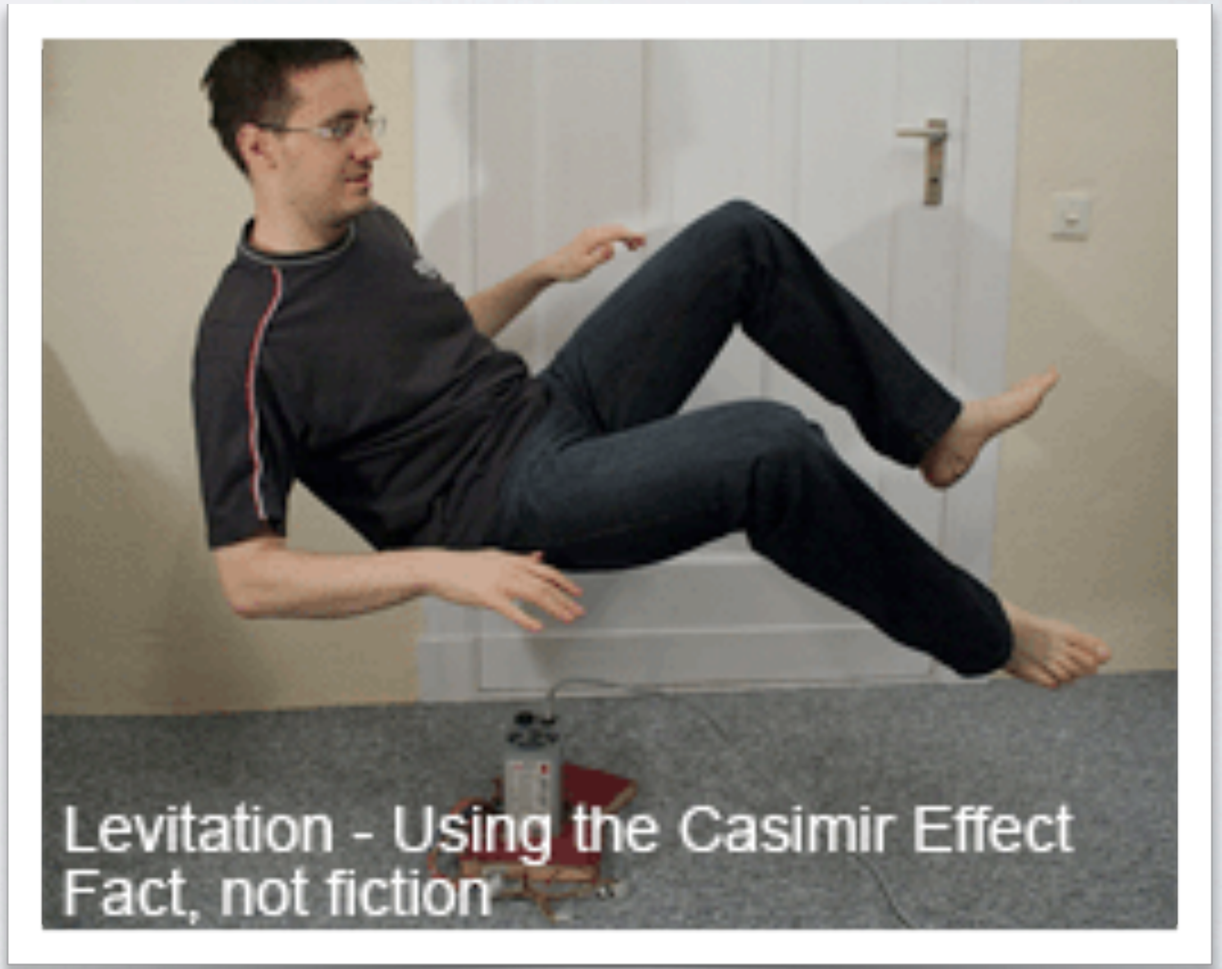
- And people can be very enthusiastic about it...



THE HISTORY OF REPULSION

- Less than 10 proposals for repulsion since 1948

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1948

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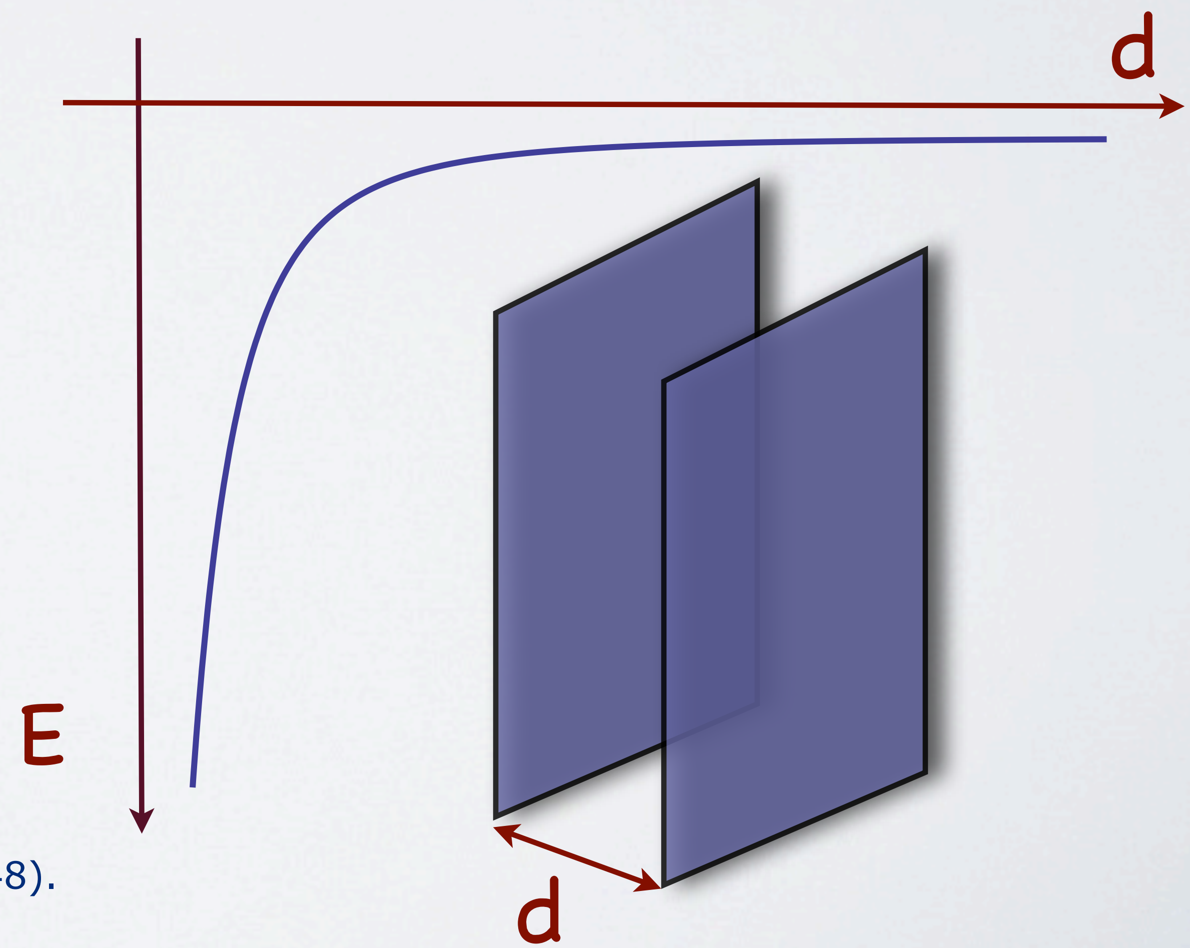

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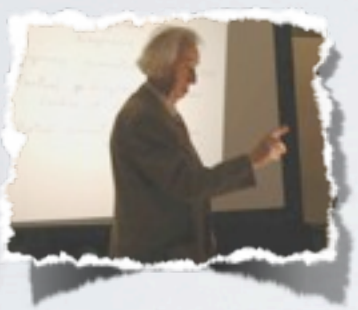
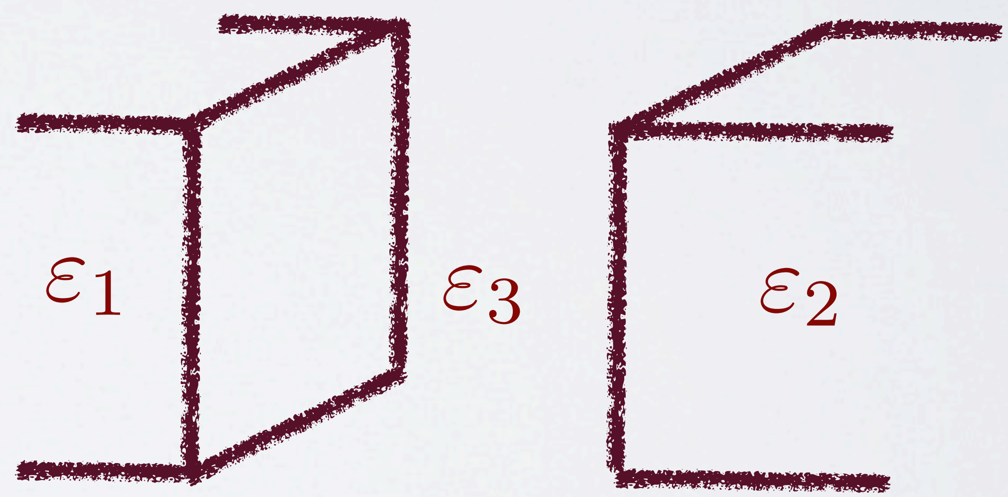


Dzyaloshinskii et al. extension to dielectrics: Repulsion under certain conditions.
No repulsion in vacuum



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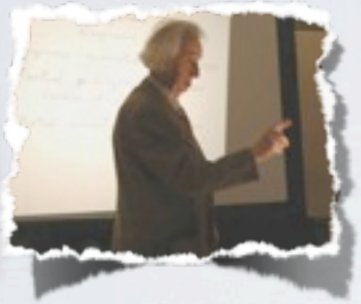
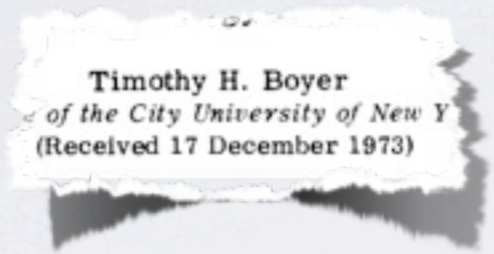
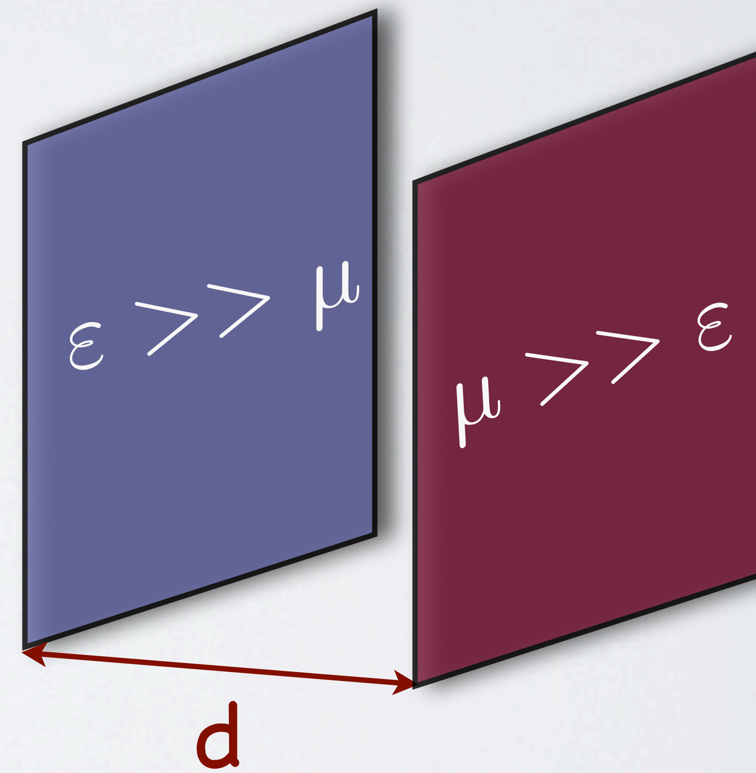



Timothy H. Boyer
of the City University of New Y
(Received 17 December 1973)

T. H. Boyer calculated the Casimir force for a perfect metallic plate and a "perfect" ($\mu \to \infty$) magnetic plate: he found Repulsion!

Boyer, Phys. Rev. A 9, 2078 (1974)



$$E_B(d) = -\frac{7}{8}E_c(d)$$

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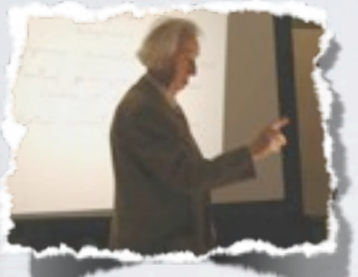

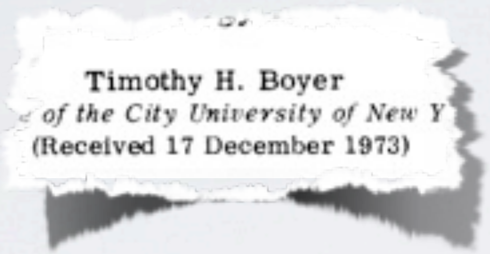
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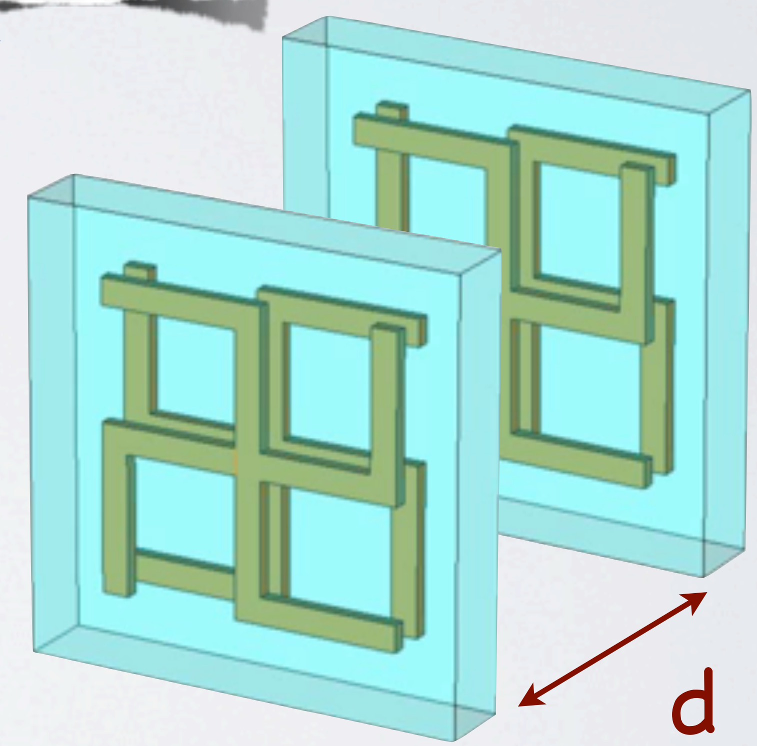
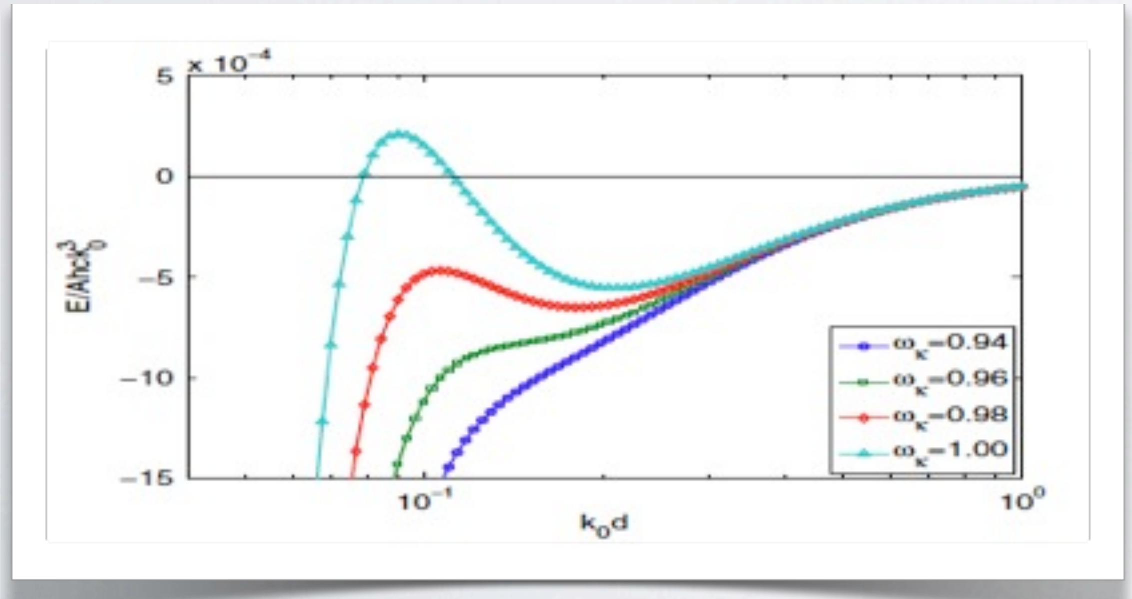
Metamaterials



Metamaterials: Several proposals that seem to give repulsion in vacuum.



Zhao *et al.* Phys. Rev. Lett 103, 103602 (2009)
U. Leonhardt and T. G. Philbin, New J. Phys. 9, 254 (2007).
F. S. S. Rosa *et al.* Phys. Rev. Lett. 100, 183602 (2008),Phys. Rev. A 78, 032117 (2008)...

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Magnetoelectric materials are excluded from the theorems. Are they helpful to obtain repulsion in vacuum?

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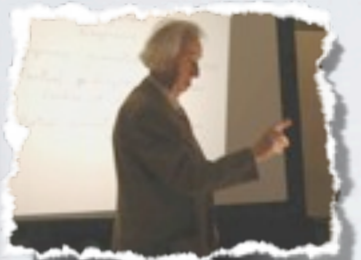
1948 1961 1974 2000's 2010 t



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Topological insulators!

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3D TOPOLOGICAL INSULATORS

- The two key ideas behind TI (both 2D and 3D):

1) Strong spin orbit coupling

2) Time reversal symmetry

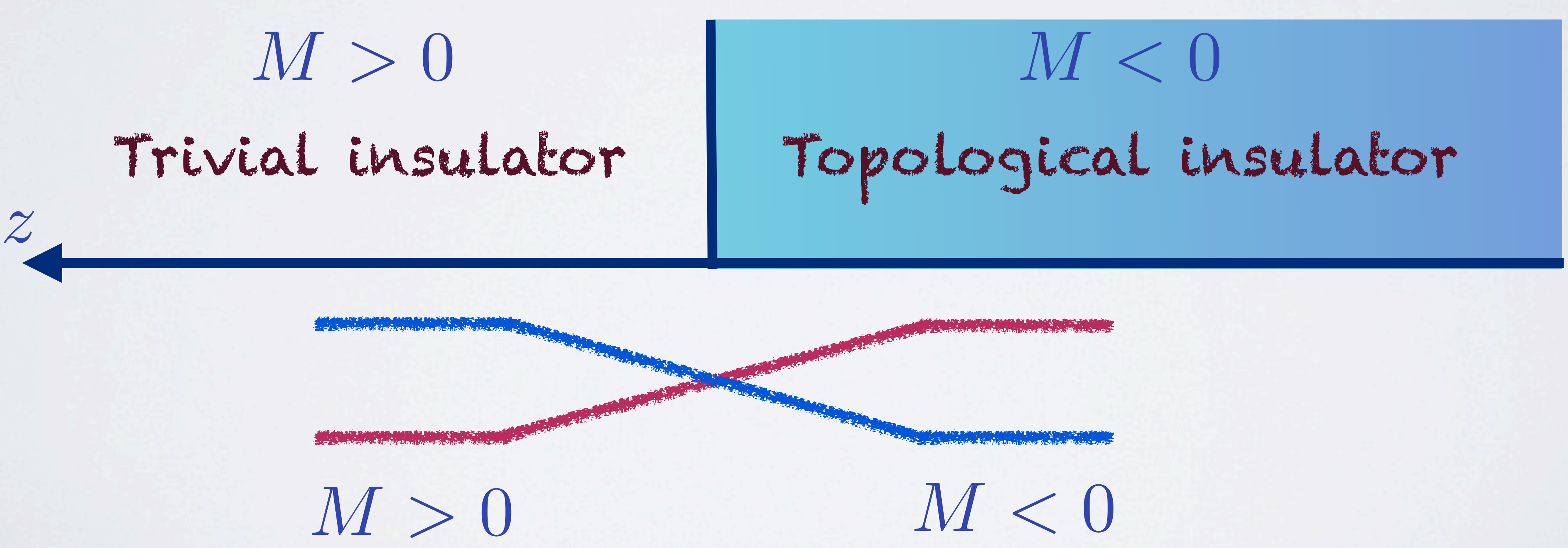
Kane, Hasan Rev. Mod. Phys, 82 3045 (2010)
Qi, Zhang arXiv:1008.2026 (2010)

3D TOPOLOGICAL INSULATORS

- The two key ideas behind TI (both 2D and 3D):

1) Strong spin orbit coupling: Band inversion (mass gap inversion)

2) Time reversal symmetry: Massless Dirac fermions on the interface



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3D TI FOR A QUANTUM FIELD THEORIST

- Start with a 3D T-invariant fermionic action describing an insulator:

$$S = \int dx^3 dt \bar{\Psi}(\not{D} - m)\Psi + F^{\mu\nu}F_{\mu\nu}$$

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$\theta = (2n+1)\pi$ TI

$\theta = 0$ Trivial

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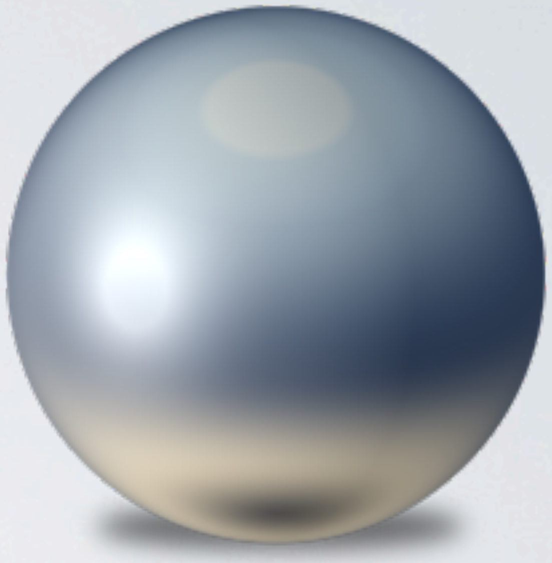

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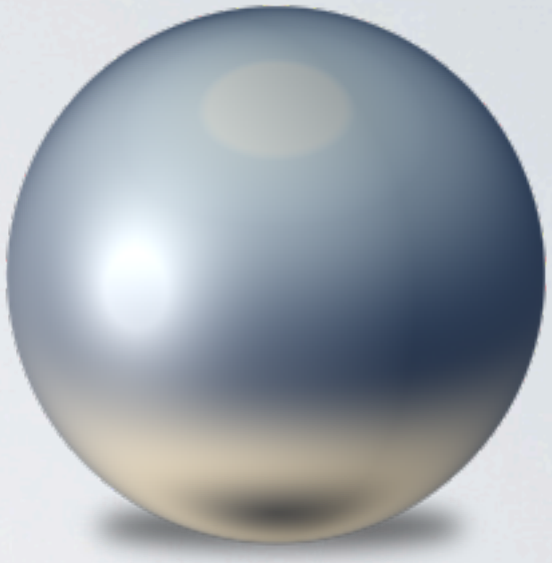

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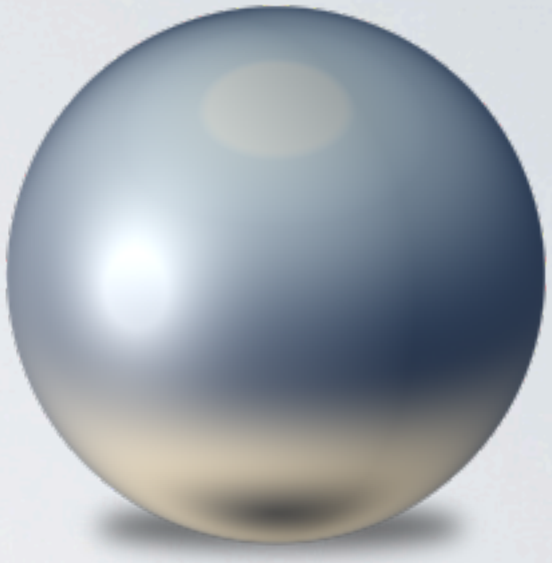

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3D TI FOR A QUANTUM FIELD THEORIST

- Fujikawa realized that the path integral measure is not invariant under a chiral transformation:

$$D\Psi^\dagger D\Psi = e^{i\int dx^3 dt\theta \frac{e^2}{8\pi h} F_{\mu\nu}\tilde{F}^{\mu\nu}} D\Psi'^\dagger D\Psi' \quad \text{where} \quad \tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

Fujikawa, Phys. Rev. Lett. **42**: 1195 (1979)
Hosur et al. PRB 81, 045120 (2010)

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Hosur et al. PRB 81, 045120 (2010)

- So the action for a T-invariant TI is finally:

$$S_{TI} = \int dx^3 dt \Psi^\dagger(\not{D} - m)\Psi + F^{\mu\nu}F_{\mu\nu} + \theta\frac{e^2}{8\pi h}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

3D TI FOR A QUANTUM FIELD THEORIST

- Fujikawa realized that the path integral measure is not invariant under a chiral transformation:

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- and the EM response is (integrating out fermions):

$$S_{TI} = \int dx^3 dt\left(\varepsilon\mathbf{E}^2 + \mu^{-1}\mathbf{B}^2\right) + \theta\frac{e^2}{2\pi h}\mathbf{E}\cdot\mathbf{B}$$

3D TI FOR A QUANTUM FIELD THEORIST

- This action is valid inside the bulk crystal

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But...the theta term implies a QHE on the boundary:

$$S_\theta = \int dx^3 dt \frac{\theta}{8\pi h} \tilde{F}^{\mu\nu} F_{\mu\nu} = \int dx^3 dt \epsilon^{\mu\nu\rho\sigma} \frac{\theta}{8\pi h} \partial_\mu \left(A_\nu \partial_\rho A_\sigma\right)$$

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QHE: $S_H = \frac{\sigma_{xy}}{2} \int dx^2 dt \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho \longrightarrow \sigma_{xy} = \frac{1}{2} \frac{e^2}{h}$

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Hence the axion term is a description of both the bulk and the boundary only when T is broken at the boundary (more on that later).

OPTICAL PROPERTIES OF 3D TI'S

- This action changes the optical response of the TI with respect to ordinary dielectrics:

$$S_{TI} = \int dx^3 dt \left(\varepsilon \mathbf{E}^2 + \mu^{-1} \mathbf{B}^2\right) + \theta \frac{e^2}{2\pi h} \mathbf{E} \cdot \mathbf{B}$$

$$= \int dx^3 dt \left(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}\right)$$

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- Maxwell's equations still valid but with different constitutive relations:

$$\boldsymbol{\nabla}\cdot\mathbf{D} = 0 \qquad \boldsymbol{\nabla}\cdot\mathbf{B} = 0$$
$$\boldsymbol{\nabla}\times\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t}$$
$$\boldsymbol{\nabla}\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$$

+

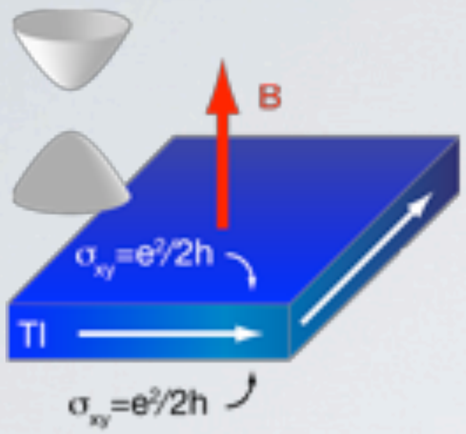
$$\mathbf{H} = \mu^{-1}\mathbf{B} - \frac{\alpha\theta}{\pi}\mathbf{E}$$
$$\mathbf{D} = \varepsilon\mathbf{E} + \frac{\alpha\theta}{\pi}\mathbf{B}$$

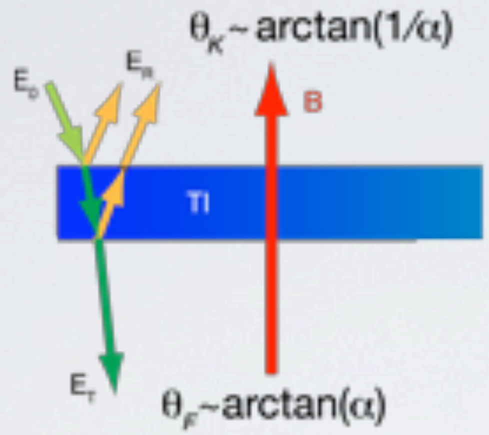
Axion electrodynamics!

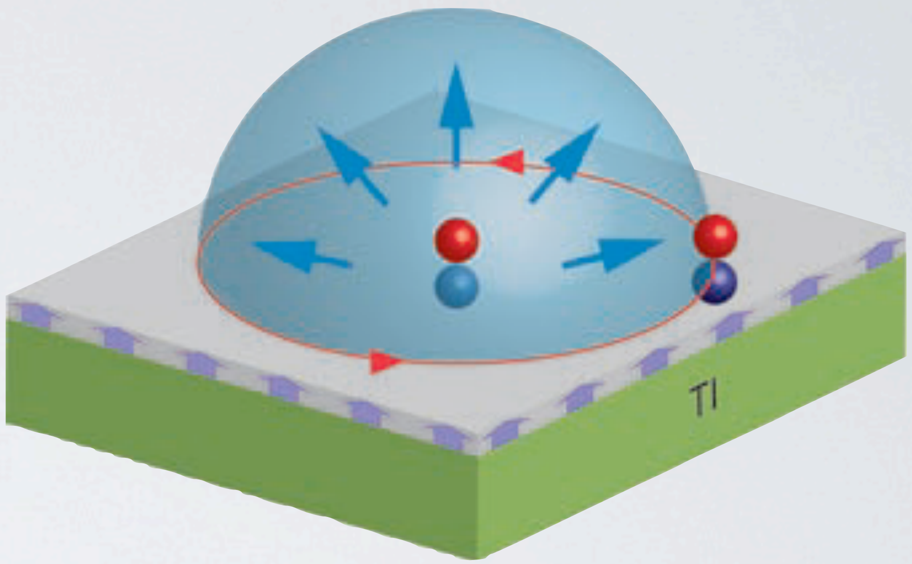
F. Wilczek, Phys. Rev. Lett. 58, 1799 (1987).

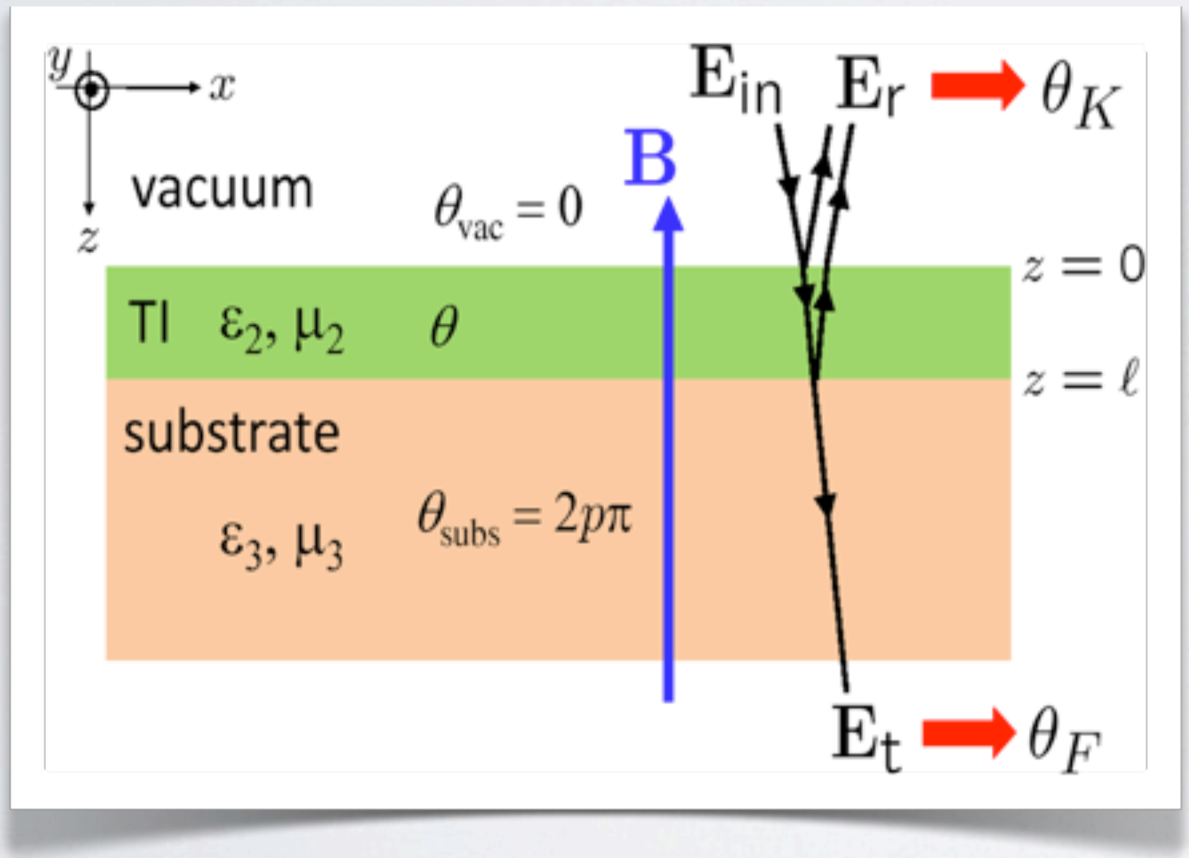
OPTICAL PROPERTIES OF 3D TI'S

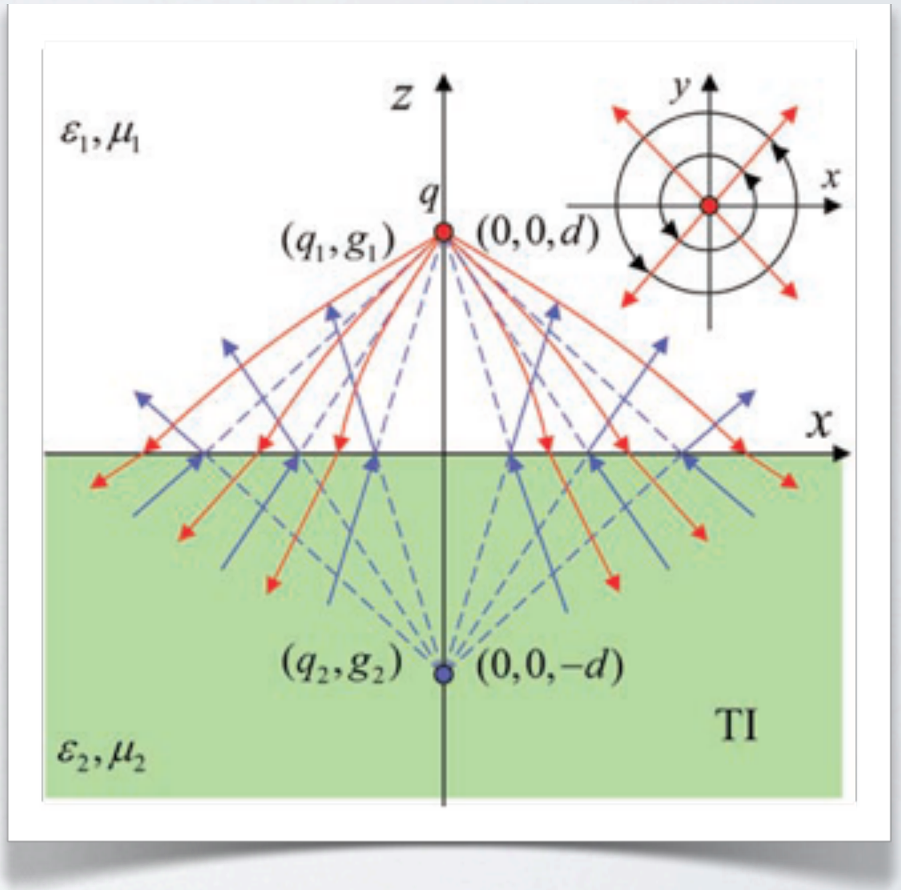
Crazy consequences of axionic physics:











Maciejko *et al.* PRL (2010), Tse *et al.* PRL (2010)

Qi *et al.* Science (2009)

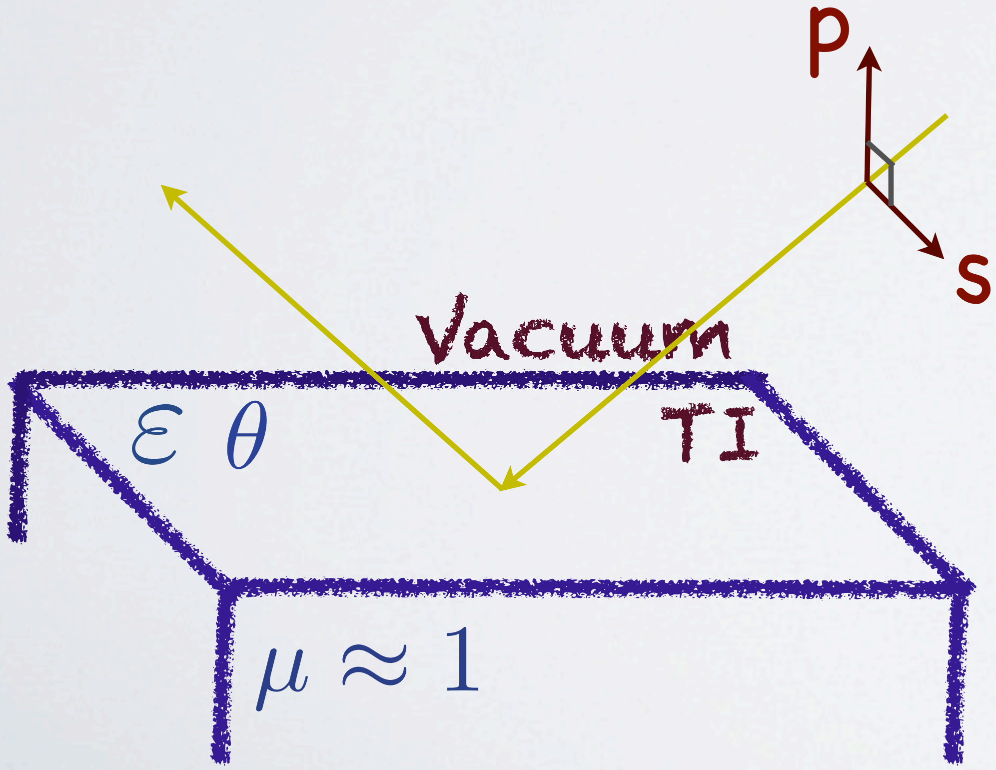
OPTICAL PROPERTIES OF 3D TI'S

1) Integrate maxwell equations to find boundary conditions:

conserved normal component: $\mathbf{D}$ and $\mathbf{B}$

conserved tangential component: $\mathbf{H}$ and $\mathbf{E}$

2) Find reflection coefficients:



$$\mathbf{R} = \left[\begin{array}{cc} R_{s,s}(i\xi, \mathbf{k}_{\|}, \theta^2) & R_{s,p}(i\xi, \mathbf{k}_{\|}, \theta) \\ R_{p,s}(i\xi, \mathbf{k}_{\|}, \theta) & R_{p,p}(i\xi, \mathbf{k}_{\|}, \theta^2) \end{array} \right]$$

with:

$$R_{s,p}(i\xi, \mathbf{k}_{\|}, \theta) = \mathrm{sgn}(\theta) r_{sp}(i\xi, \mathbf{k}_{\|}, |\theta|)$$

Chang *et al.* Phys. Rev. B 80, 113304 (2009)

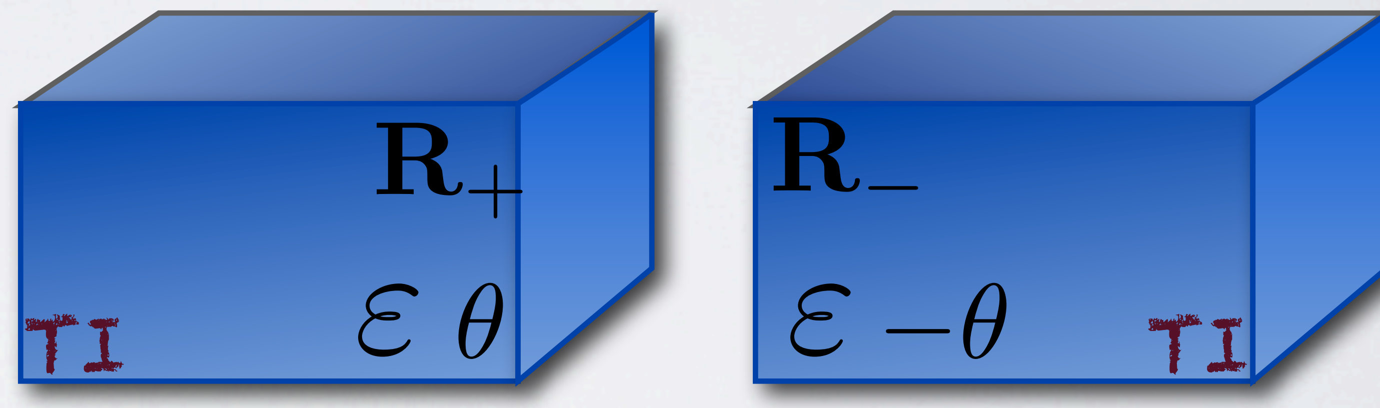
LIFSHITZ FORMULA FOR TI'S

- Can we prove analytically that there will be repulsion?

$$\frac{E_c(d)}{A\hbar} = \int_0^\infty \frac{d\xi}{2\pi} \int \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2} \log\det\left[1 - \mathbf{R}_1 \cdot \mathbf{R}_2 e^{-2k_3 d}\right]$$

Consider the case:

$$\theta_1 = -\theta_2 \equiv \theta$$

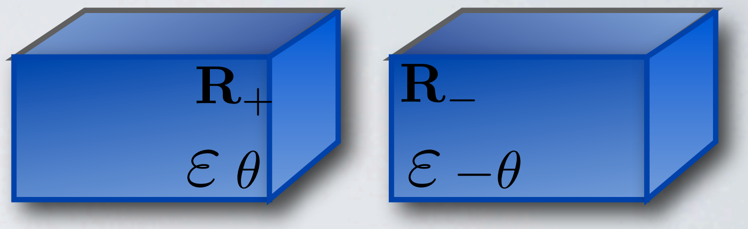


$$\mathbf{R}_\pm = \begin{bmatrix} r_s(i\xi, \mathbf{k}_\parallel, \theta^2) & \pm r_{sp}(i\xi, \mathbf{k}_\parallel, |\theta|) \\ \pm r_{sp}(i\xi, \mathbf{k}_\parallel, |\theta|) & r_p(i\xi, \mathbf{k}_\parallel, \theta^2) \end{bmatrix}$$

LIFSHITZ FORMULA FOR TI'S

Integrand of the Lifshitz formula at long and short distances:

$$I \approx \frac{1}{d^3} \log \left[1 + e^{-2k_3^{(r)}} \left(2r_{sp}^2 - r_p^2 - r_s^2 \right) \right],$$



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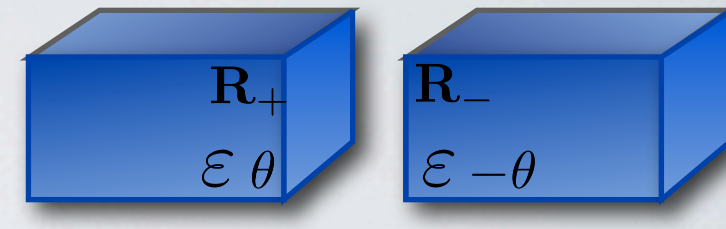
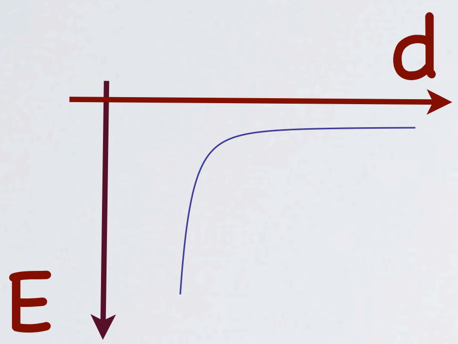


$\underline{d \to \infty}$ $\qquad r_{sp} < r_p \qquad I \approx \frac{1}{d^3} \log \left[1 - B^2 \right] < 0$



the integrand goes to zero from negative values

Attraction!

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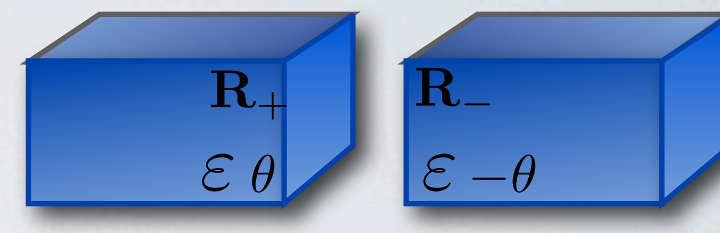

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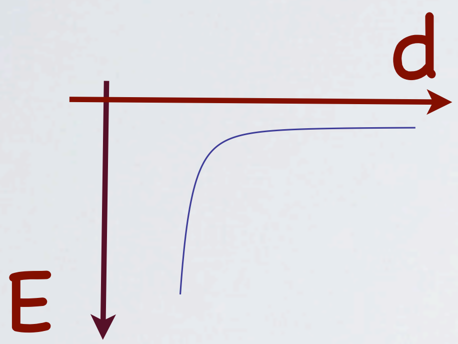

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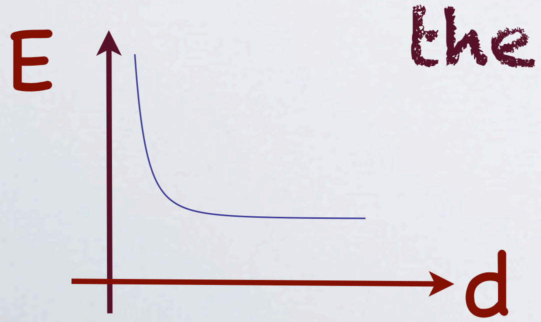
Attraction!

$\underline{d \to 0}$ $\qquad r_{sp} > r_p \qquad I \approx \frac{1}{d^3}\log\left[1 + B'^2\right] > 0$

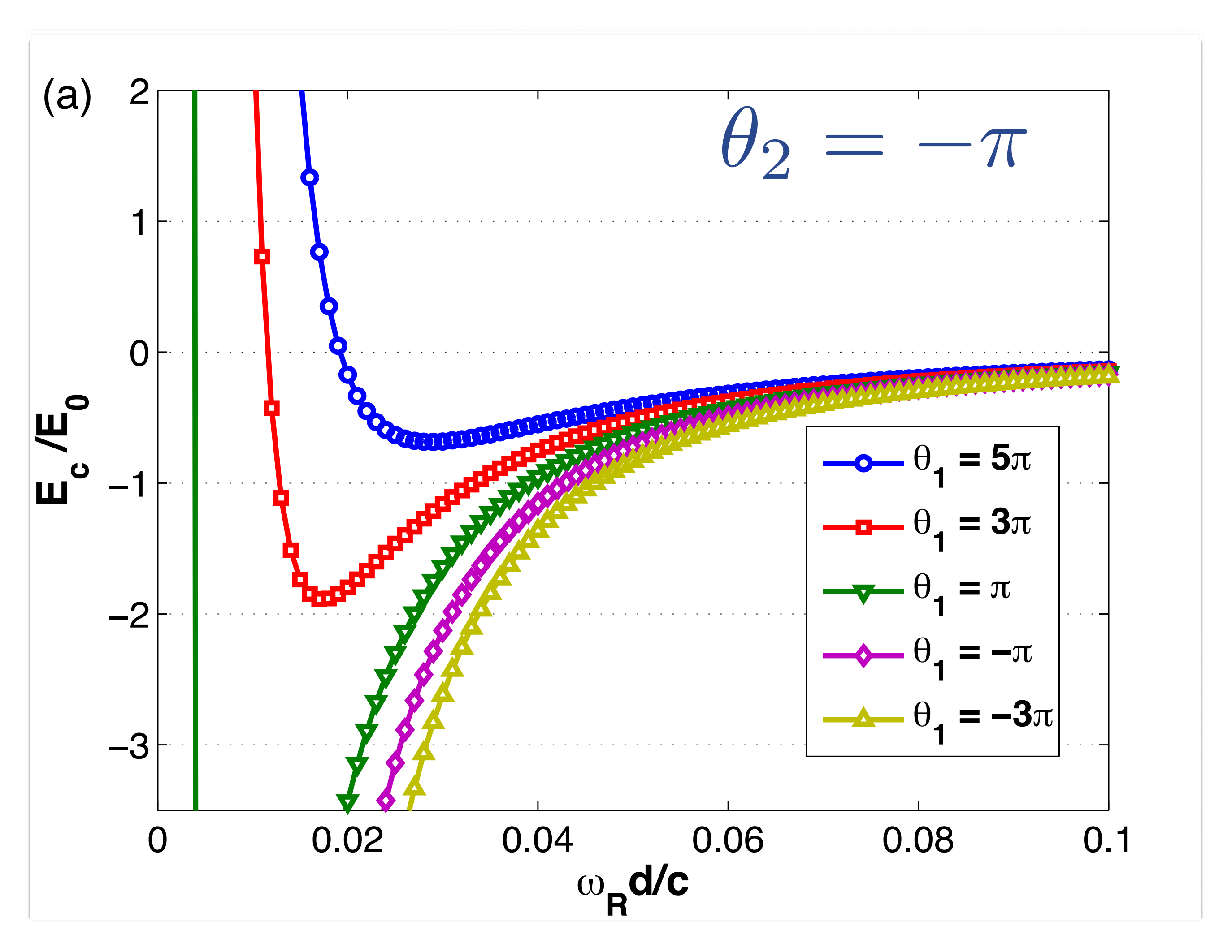


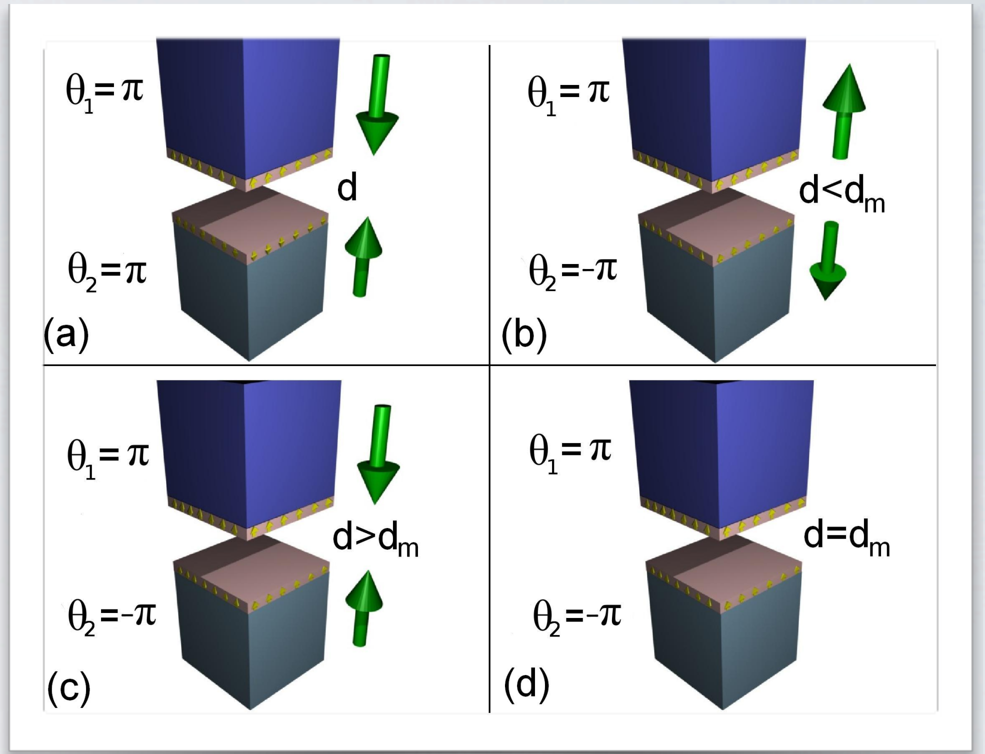
the integrand goes to infinity from positive values

Repulsion!

NUMERICAL INTEGRATION:

Analytical analysis confirmed by numerics:



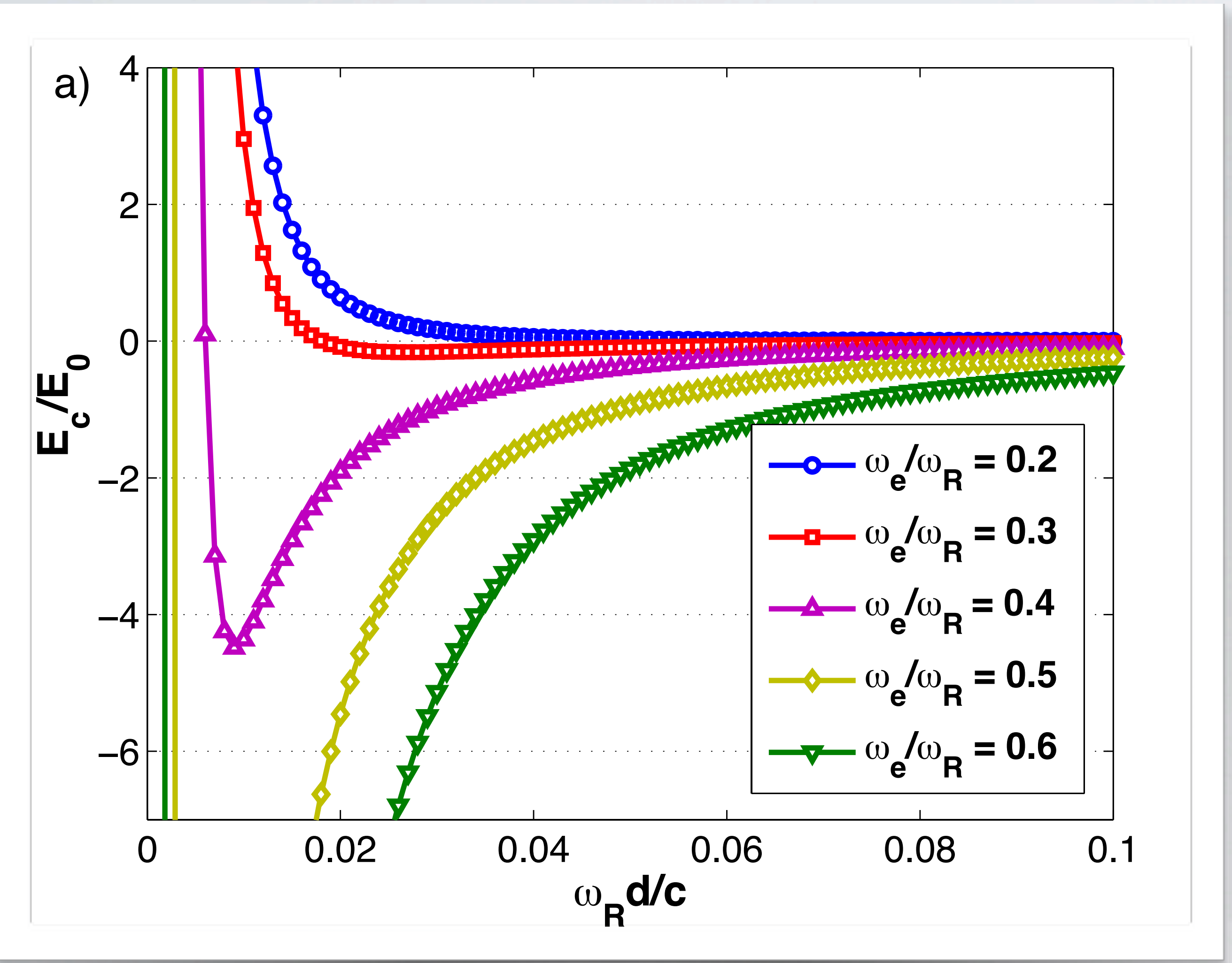


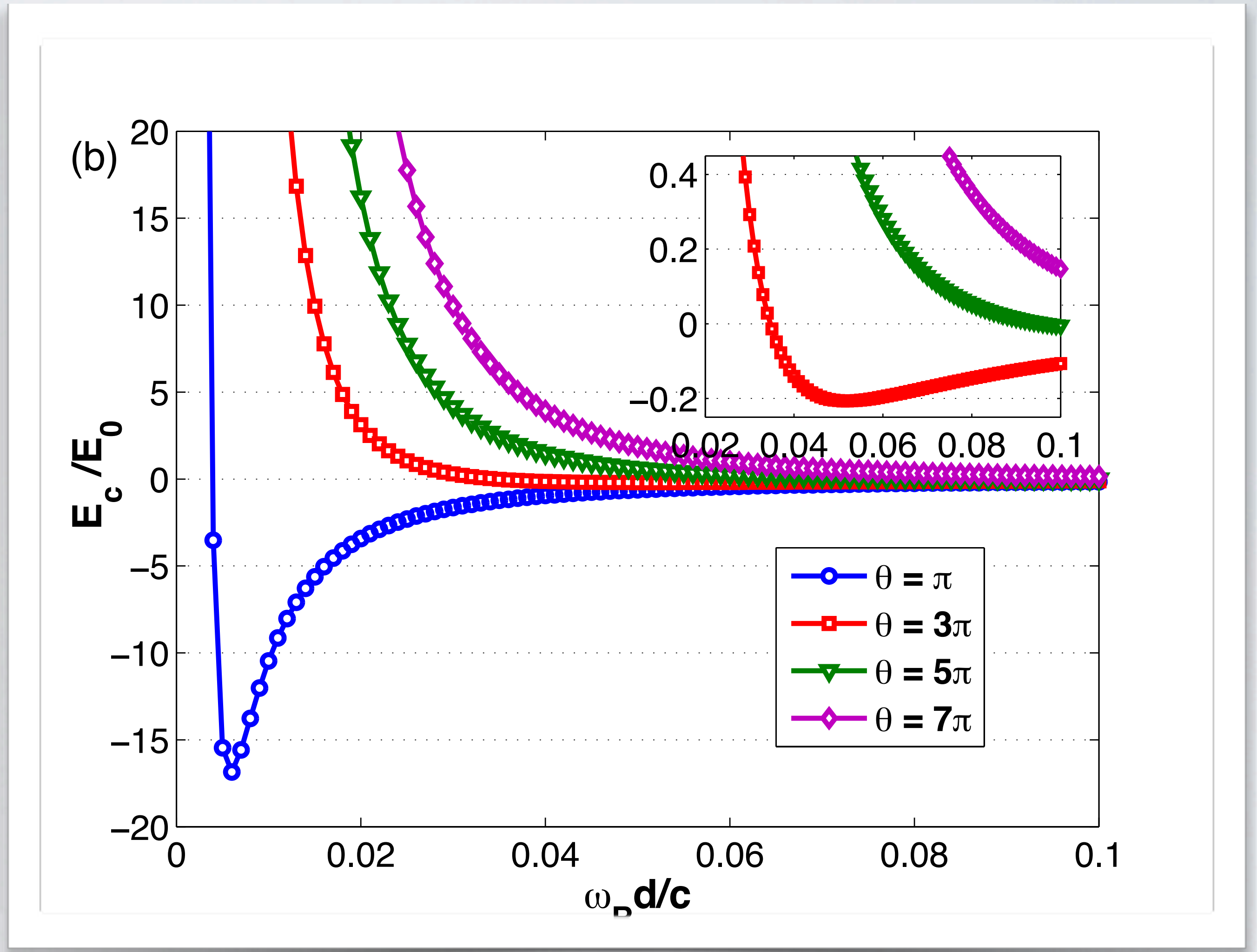
Opposite signs for θ terms generate repulsion at short distances

At short distances, the magnetoelectric coupling wins, while at large distances, it is the ordinary dielectric response.

NUMERICAL INTEGRATION:

Dependency on the parameters:





Low $\varepsilon(0)$ favors repulsion at lower distances.

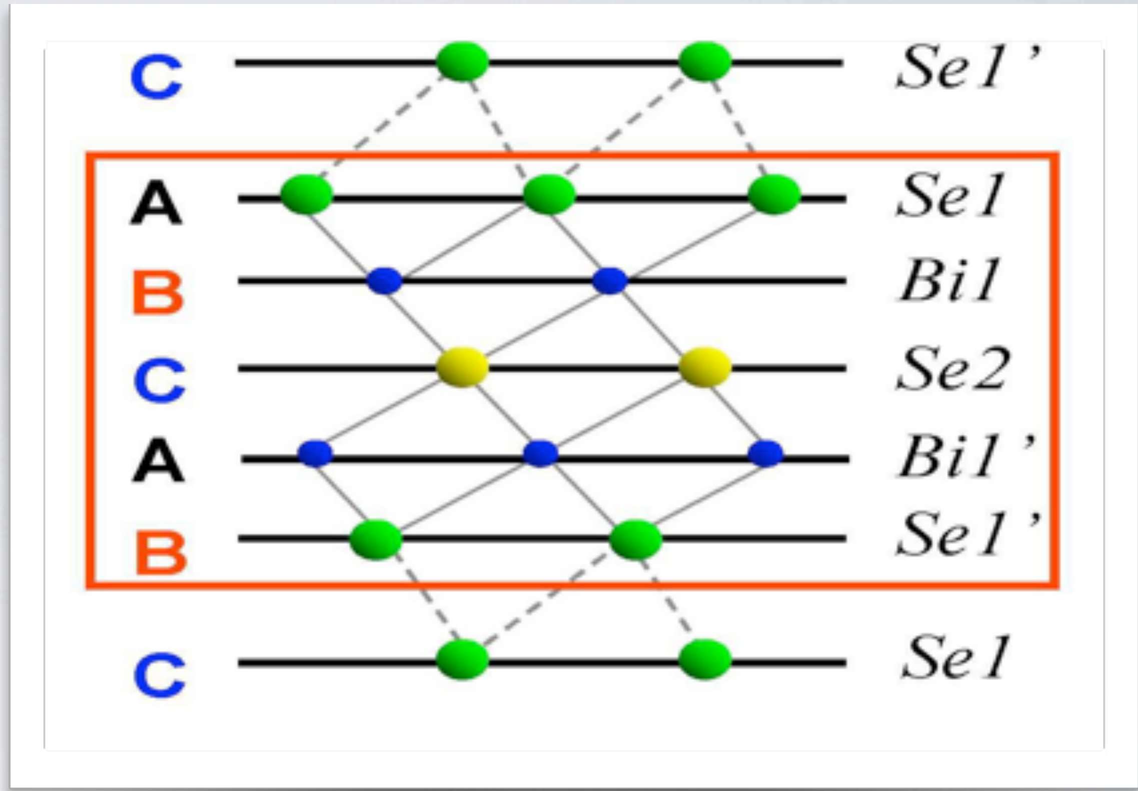
High θ favors repulsion at lower distances

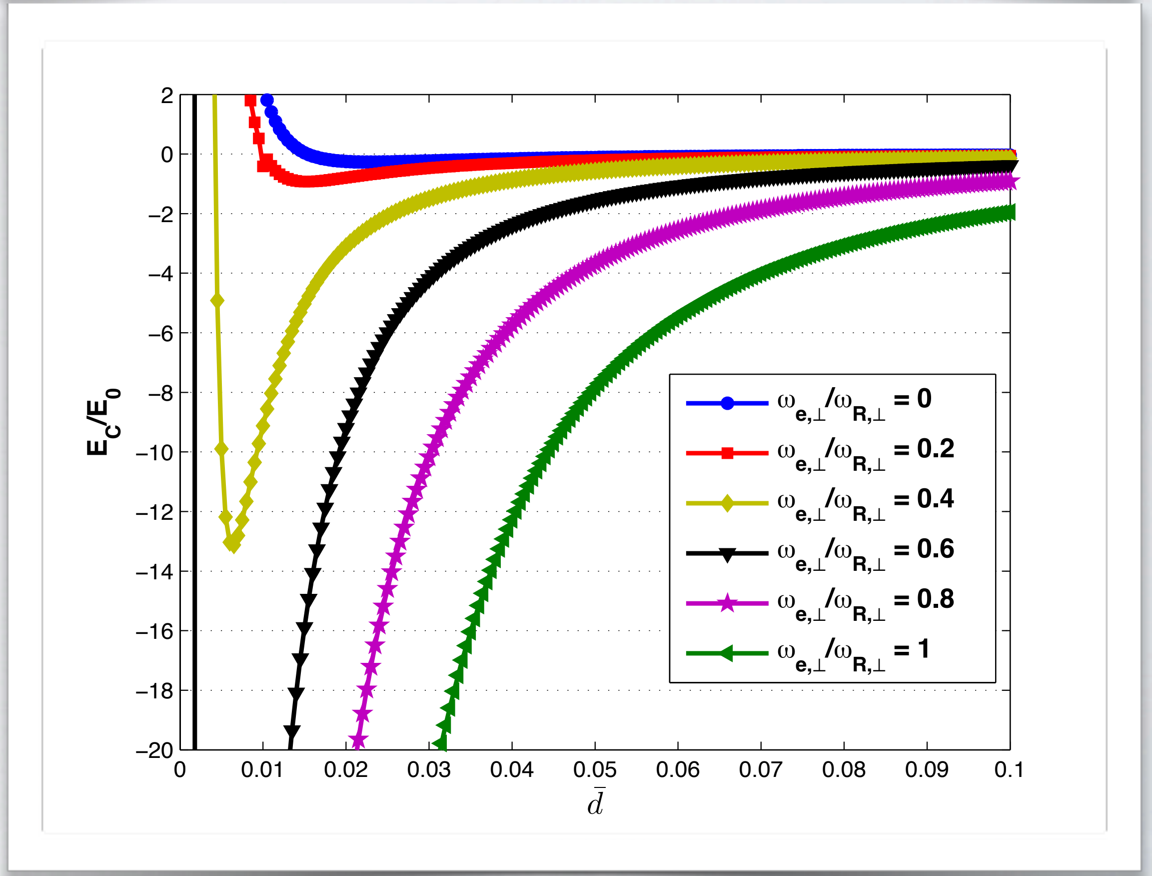
WHAT ABOUT REAL MATERIALS?

Real TI are anisotropic
(all uniaxial for the moment)

$$\varepsilon(\omega)_{ij} = \mathrm{diag}(\varepsilon_\perp, \varepsilon_\perp, \varepsilon_\parallel)$$

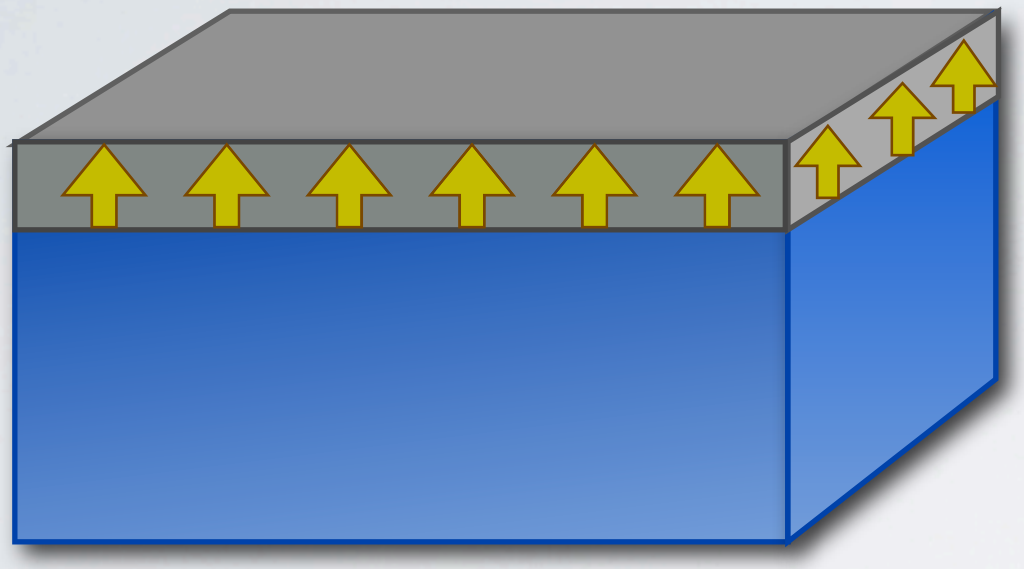
Anisotropy can enhance repulsion if $\varepsilon_\parallel > \varepsilon_\perp$ for a wide range of frequencies





WHAT ABOUT REAL MATERIALS?

T must be broken at the surface: Insulating ferromagnet opens a gap through Zeeman coupling at the surface.



Effective surface hamiltonian:

$$\mathcal{H}_s = \boldsymbol{\sigma} \cdot |\mathbf{k}| + M\sigma_z$$

$$\sigma_{xy} = \text{sign}(M)\frac{e^2}{2h}$$

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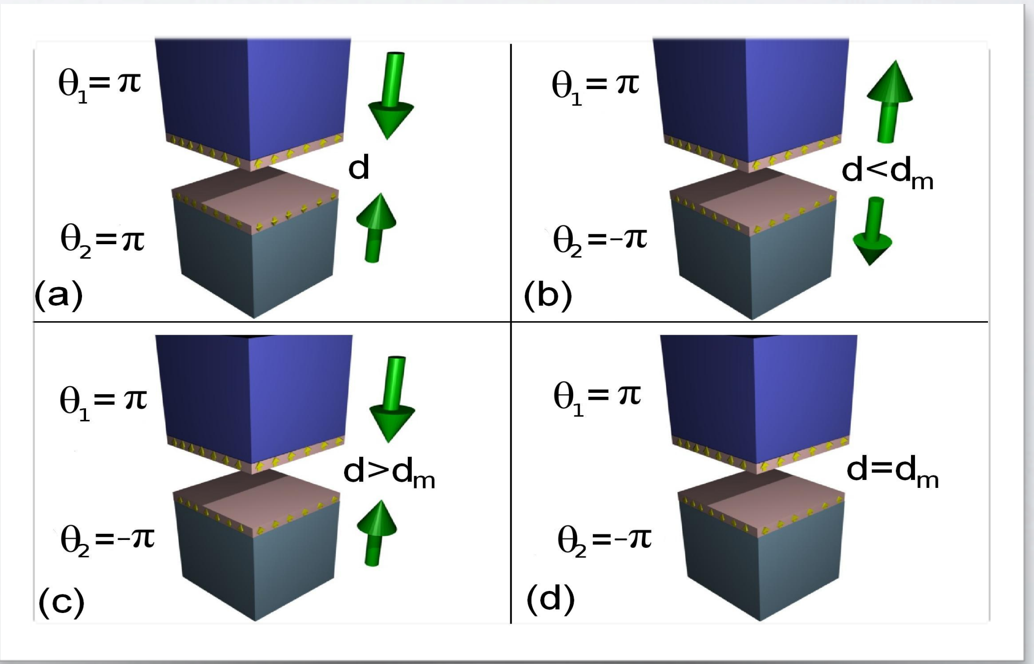
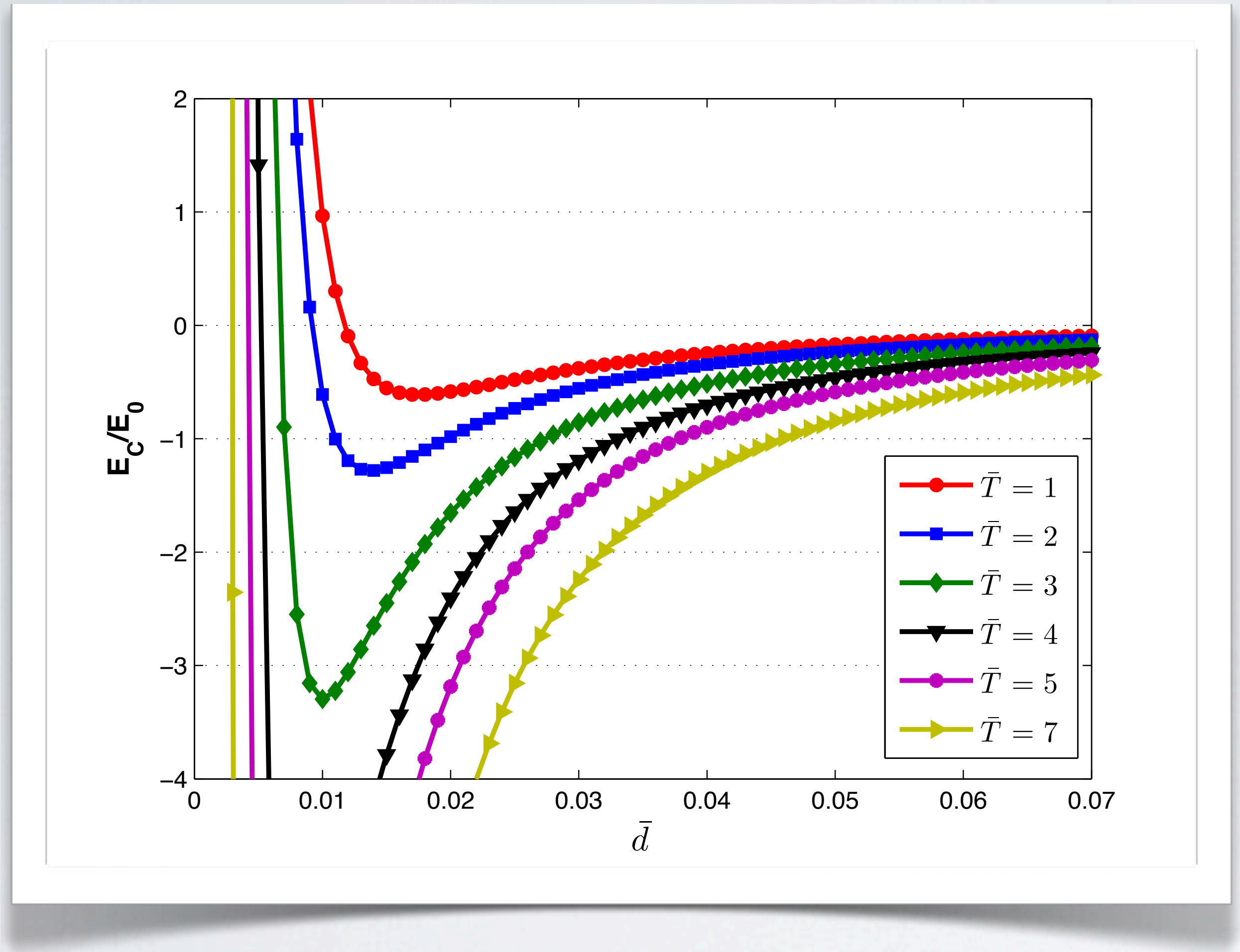
The sign of the magnetization controls the QHE of the surface and thus the sign of theta.

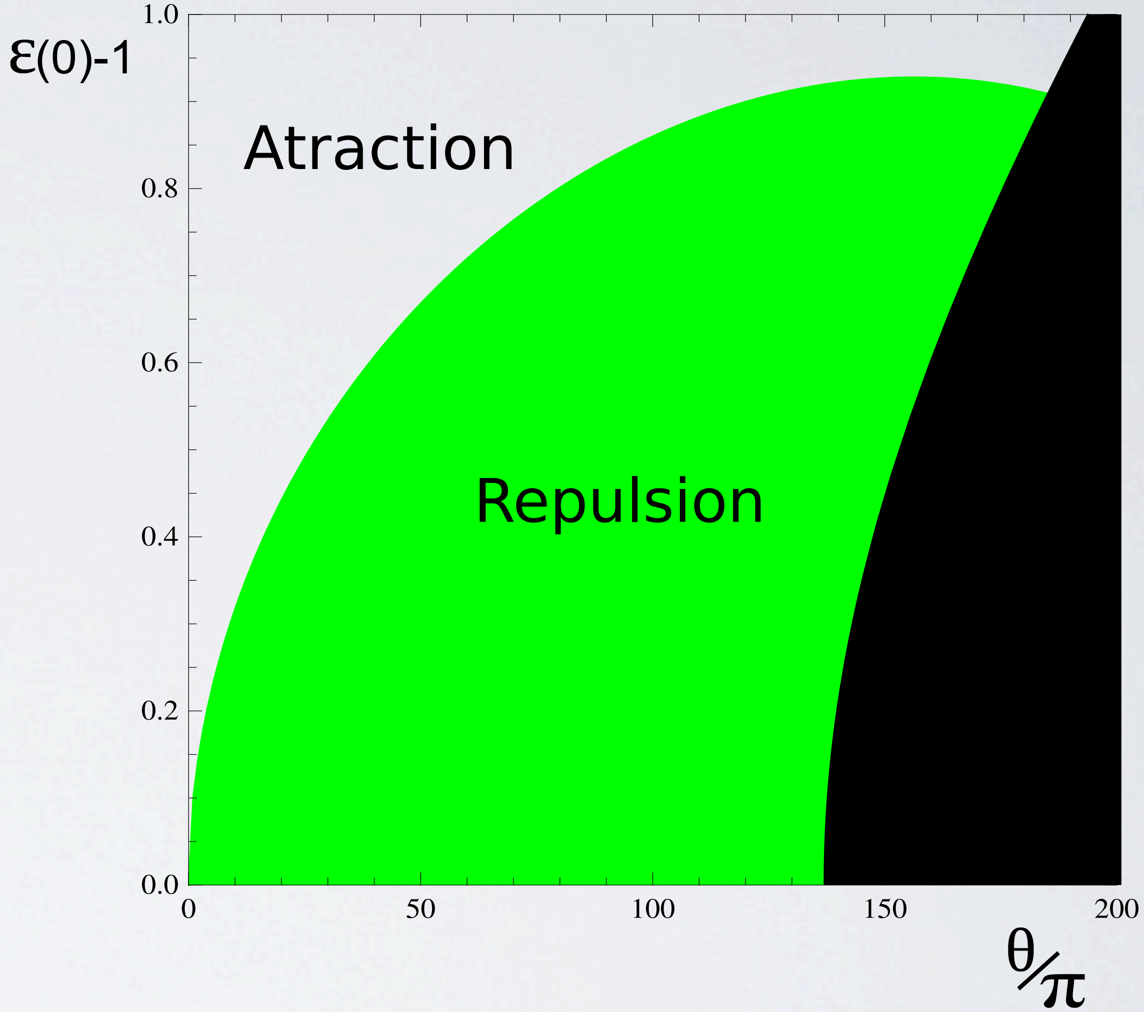
Controlling M controls the Casimir response!



WHAT ABOUT TEMPERATURE?

Temperature plays against repulsion:





Finite T

$T \to \infty$

CONCLUSIONS:

Casimir effect with TI is very unusual and could be in principle even tunable with external parameters

Anisotropy can be used to enhance the effect, but temperature plays against repulsion

Magnetoelectric materials still remain an unexplored area in Casimir physics, and could be used to achieve repulsion in vacuum.

http://focus.aps.org/story/v27/st1

APS physics Physical Review Focus

A. G. G., A. Cortijo, Phys. Rev. Lett. 106, 020403 (2011)

A. G. G., P. Rodriguez-Lopez, A. Cortijo, arXiv: 1102. 0455 Phys. Rev. B in press