CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
INSTITUTO DE CIENCIA DE MATERIALES DE MADRID (ICMM)

Centro de Física do Porto

Topological Fermi Liquids in the doped Honeycomb lattice

Eduardo V. Castro[1, 2]

Adolfo G. Grushin[1], Belén Valenzuela[1], María A. H. Vozmediano[1]
Alberto Cortijo[3], Fernando de Juan[4]

[1] Instituto de Ciencia de Materiales de Madrid, CSIC, Spain
[2] Centro de Física do Porto, UP, Portugal
[3] Universidad Autónoma de Madrid, Spain
[4] Indiana University, USA

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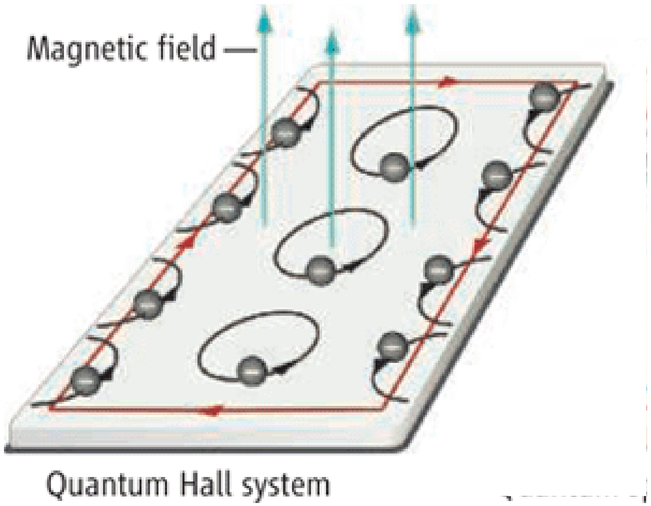
Outline

- *Motivation / Introduction*
  - Topological metals and Key ingredients
- *How to get a $\mathcal{T}$- broken phase*
  - Historical example
  - Interactions as a route
  - Interactions in an **enlarged unit cell**
- *Case study: the honeycomb lattice*
  - Phase diagram
  - $\mathcal{T}$- broken phases
  - Finite AH conductivity
- *Conclusions*

Topological phases I

- *Phases of matter described by topological invariants* (rather than order parameters and broken symmetries)
  - Known examples:

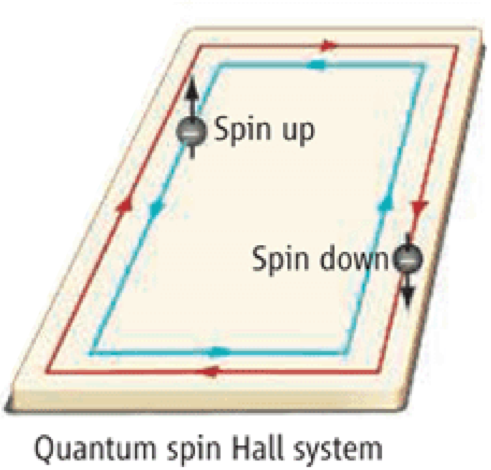
Quantum Hall insulators



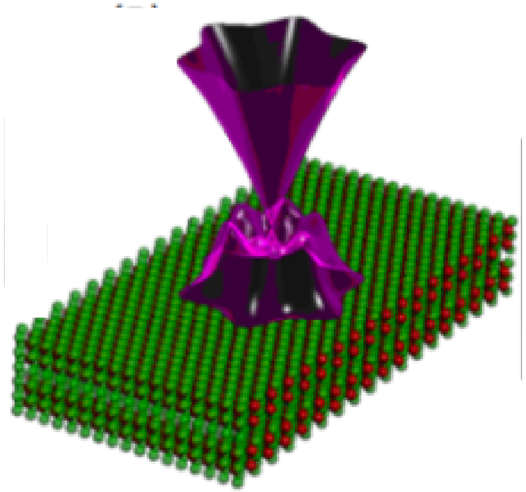
T - broken

Topological insulators

2D



3D



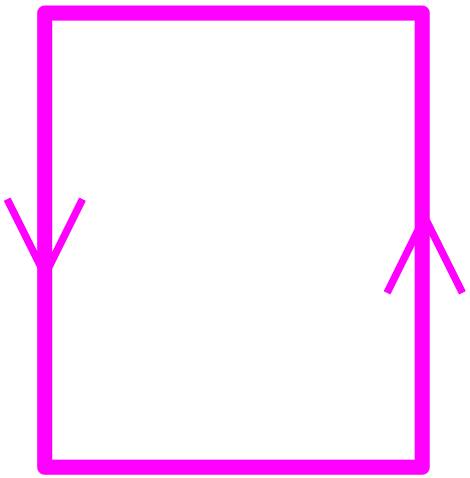
T – not broken

(spin is crucial)

Topological phases II

- *Quantum Anomalous Hall phases*
  - Like QH insulators without magnetic field

B=0



<u>Key ingredients:</u>

- Time reversal symmetry $(\mathcal{T})$ broken
- Non-trivial Bloch wave function topology

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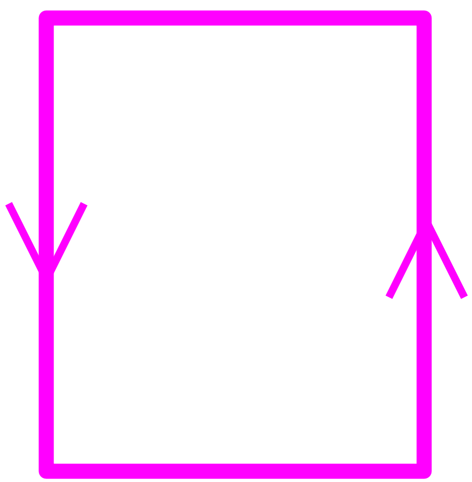

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Quantitatively: $$\sigma_0^{ab}(\mu) = \frac{e^2}{\hbar}\frac{1}{\Omega N}\sum_{kn}\mathcal{F}_n^{ab}\, n_n^0(\mathbf{k},\mu)$$

Fermi-Dirac

Berry curvature

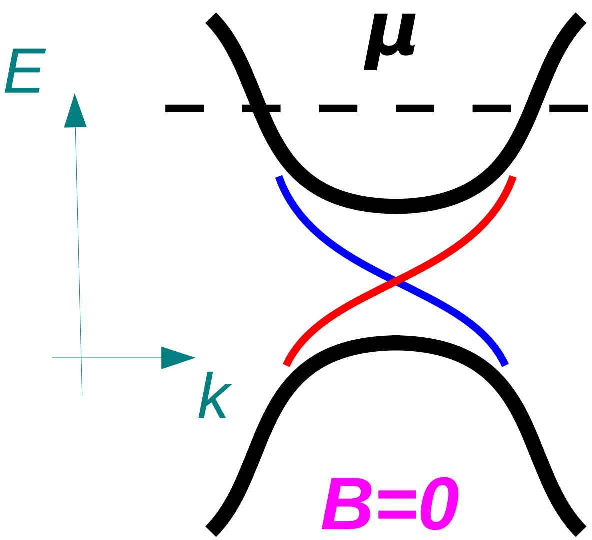
$$\mathcal{F}_n^{ab}(\boldsymbol{k}) = \nabla_k^a \mathcal{A}_n^b(\boldsymbol{k}) - \nabla_k^b \mathcal{A}_n^a(\boldsymbol{k}) \qquad \mathcal{A}_n^a(\boldsymbol{k}) = -i\langle\psi_n(\boldsymbol{k})|\nabla_k^a\psi_n(\boldsymbol{k})\rangle$$

Berry connection

Bloch wave function

Topological phases III

- *Anomalous Hall (metallic) phases*
  - Classical Hall effect without magnetic field



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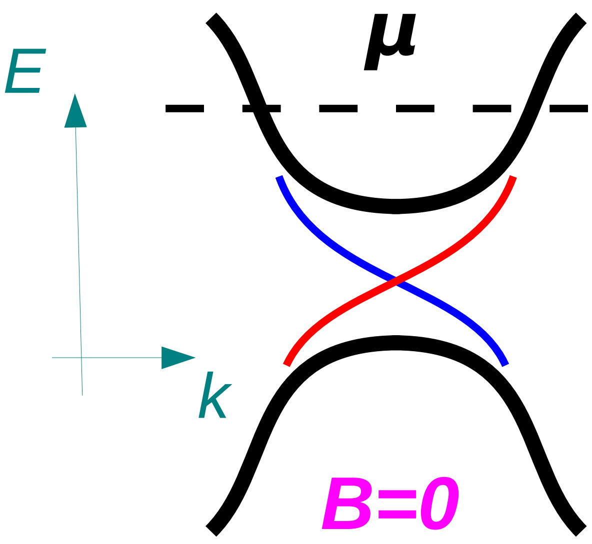

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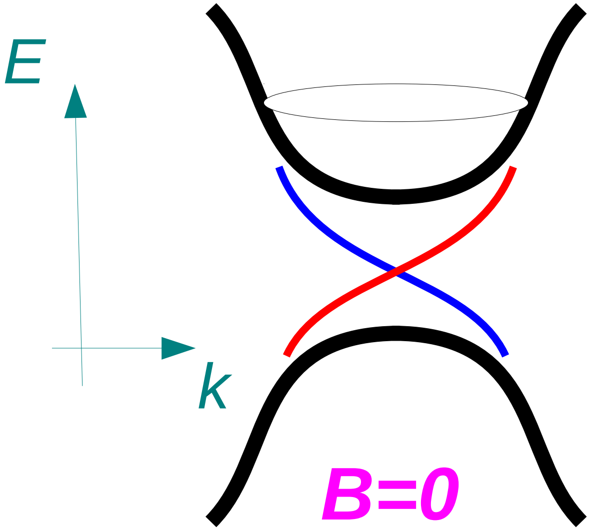

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Haldane
PRL, (2004)

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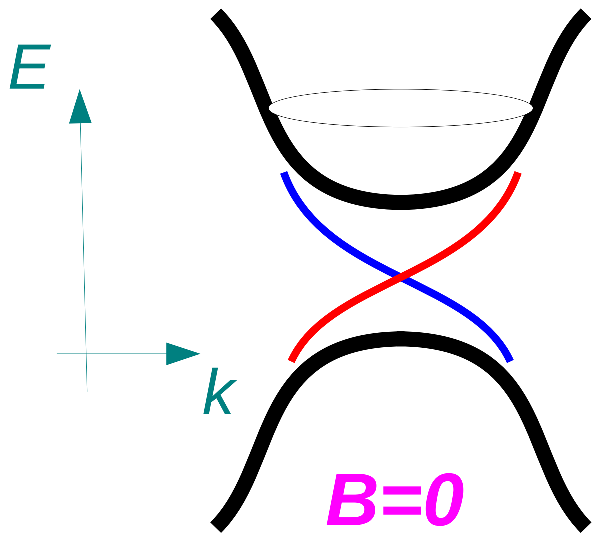

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Challenging to find simple models (or not so simple) of topologically non trivial $\mathcal{T}$ – broken phases

Sun & Fradkin
PRB, (2008)

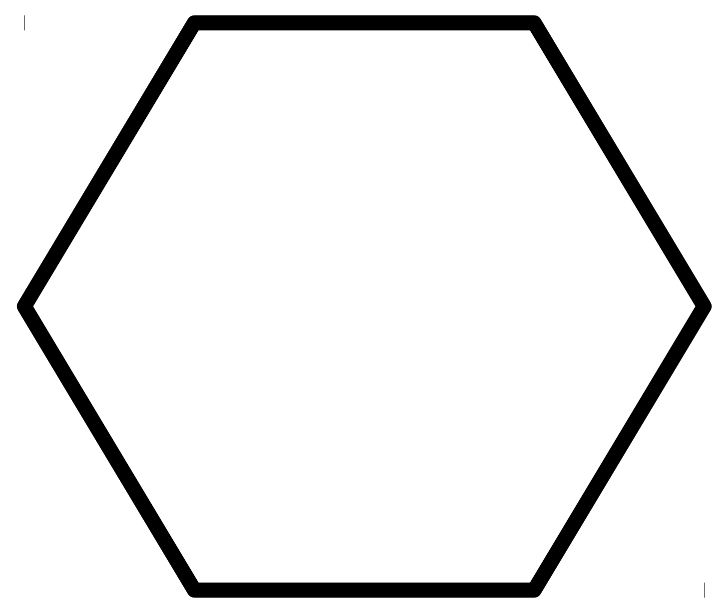
How to break T?

- *Historical example*
  - (Add imaginary hoppings in a clever way)

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane



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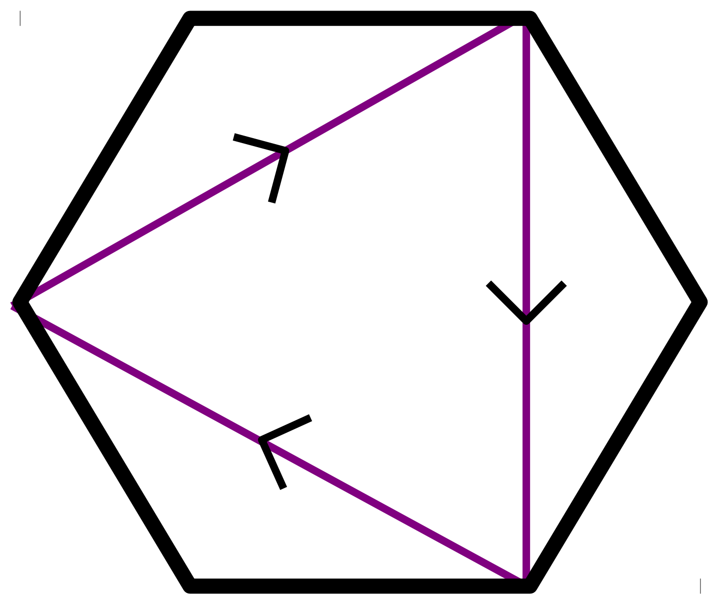

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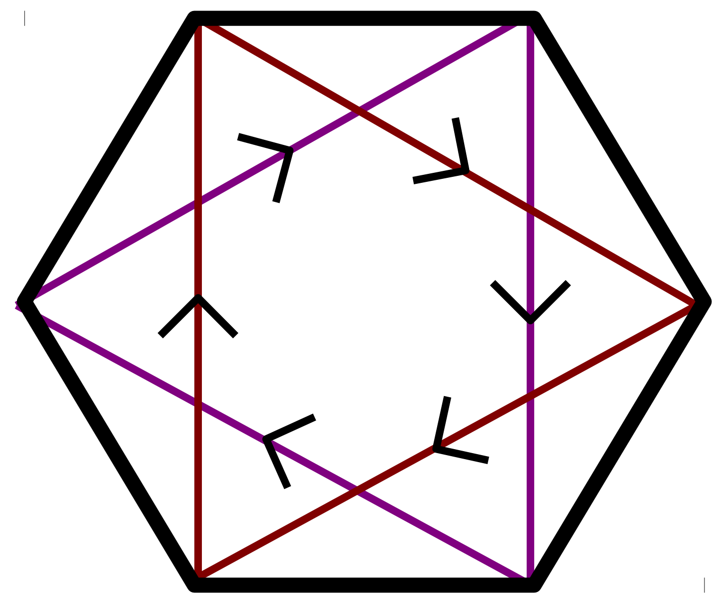

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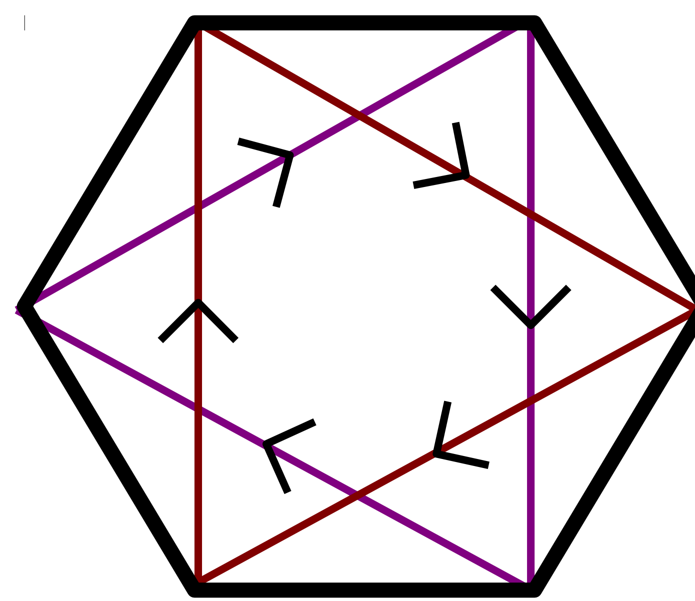
F. D. M. Haldane

- **QAH phase at half filling**

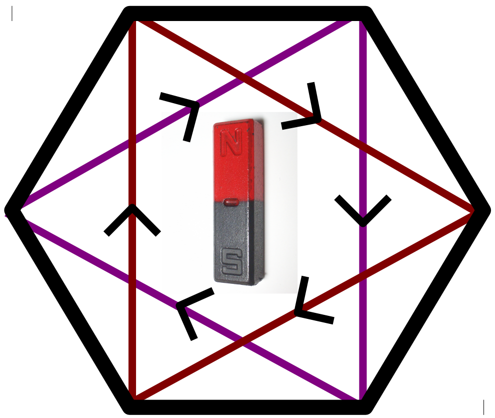
- **Anomalous Hall phase when doped**



Realizations of Haldane model

- *Original Haldane's proposal*
  - Magnetic dipole at the hexagon center

    (experimental realization ??)



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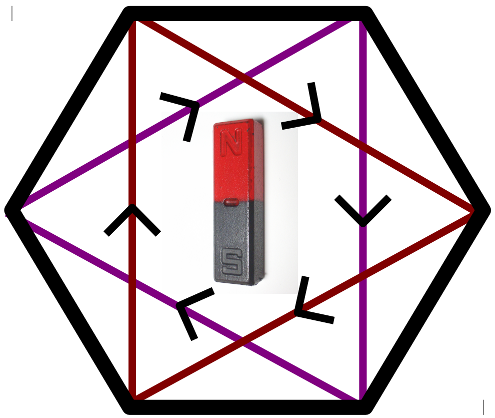
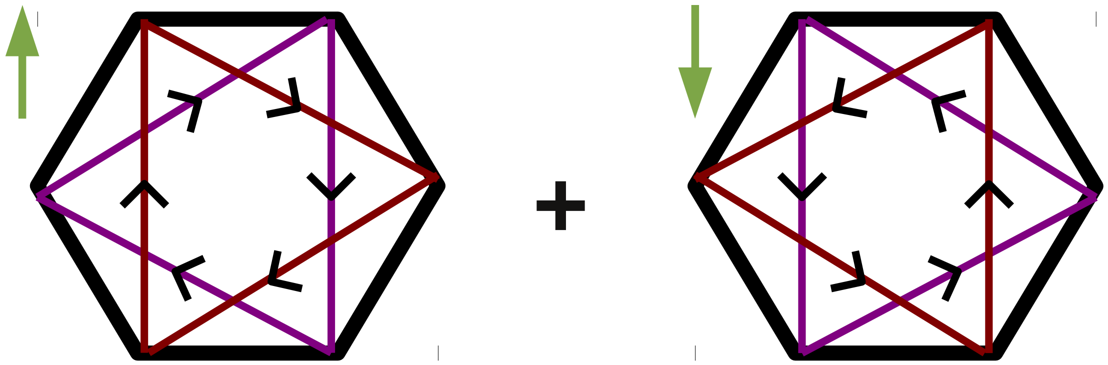


- *Spin orbit coupling*
  - Two copies of Haldane model

Kane & Mele
PRL, (2005)



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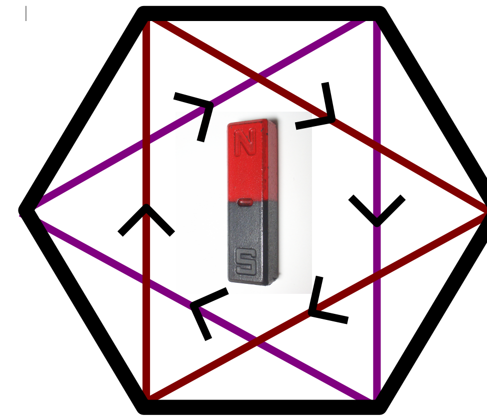

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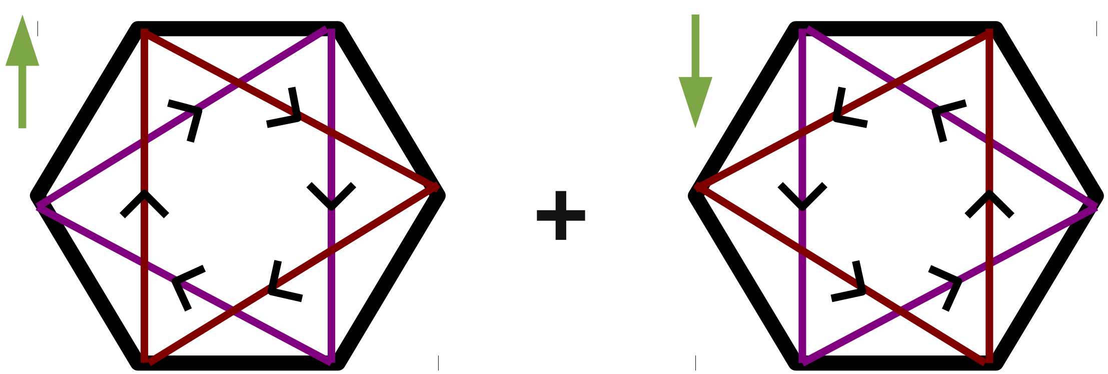
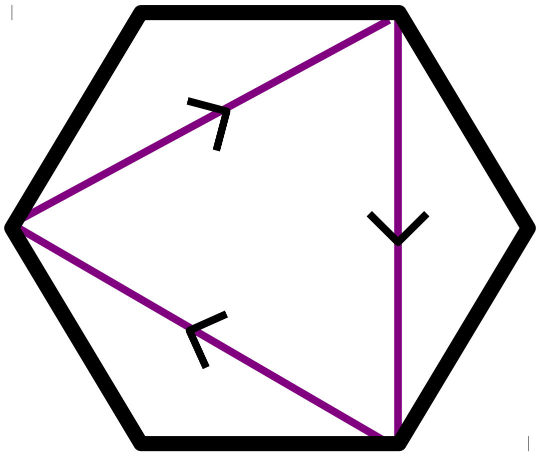


QSH effect
$\mathcal{T}$ *not broken*

What about correlations?

Alternative route: Interactions

- *Key:* ***self-consistent orbital currents***
  - Raghu *et al.* PRL, (2008)



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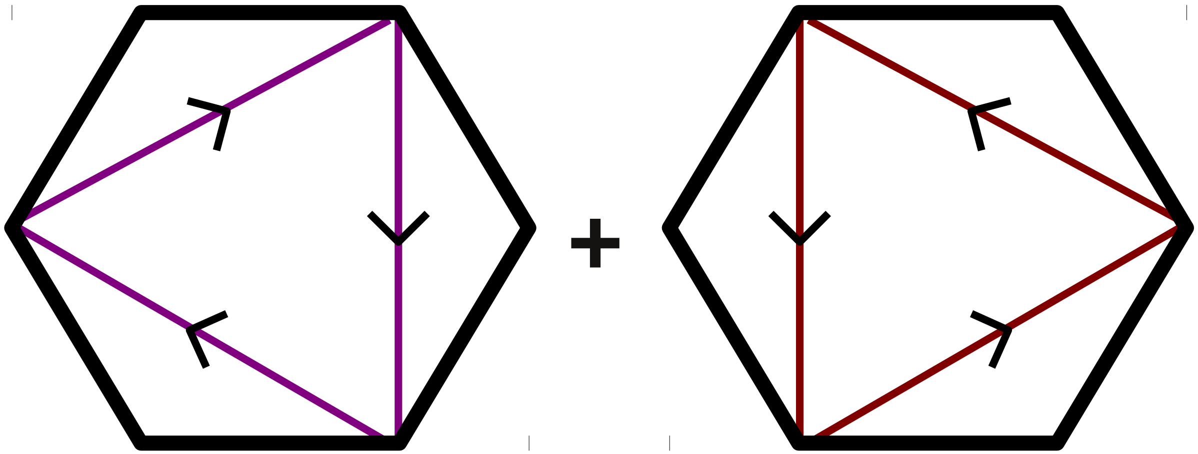

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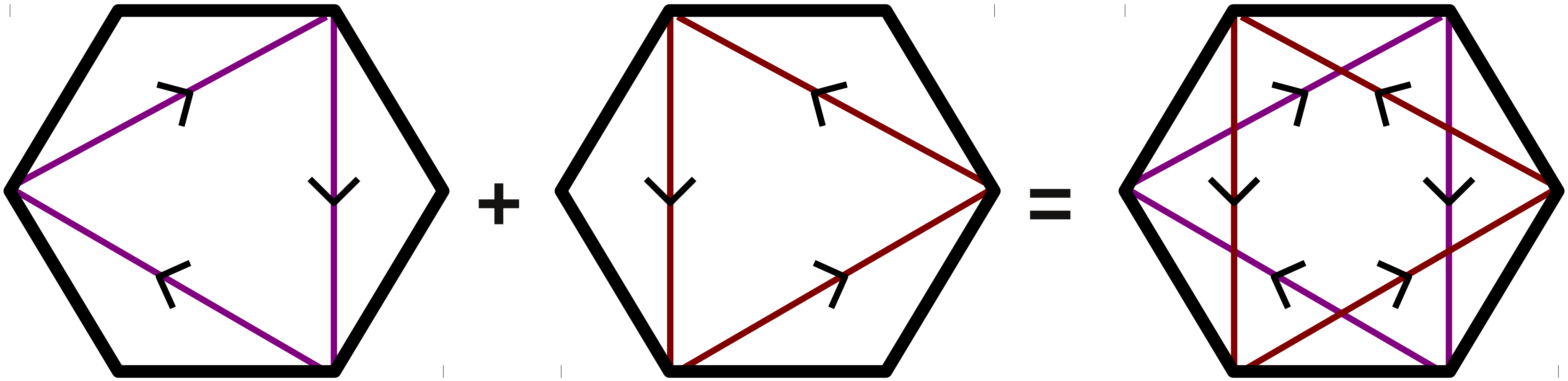

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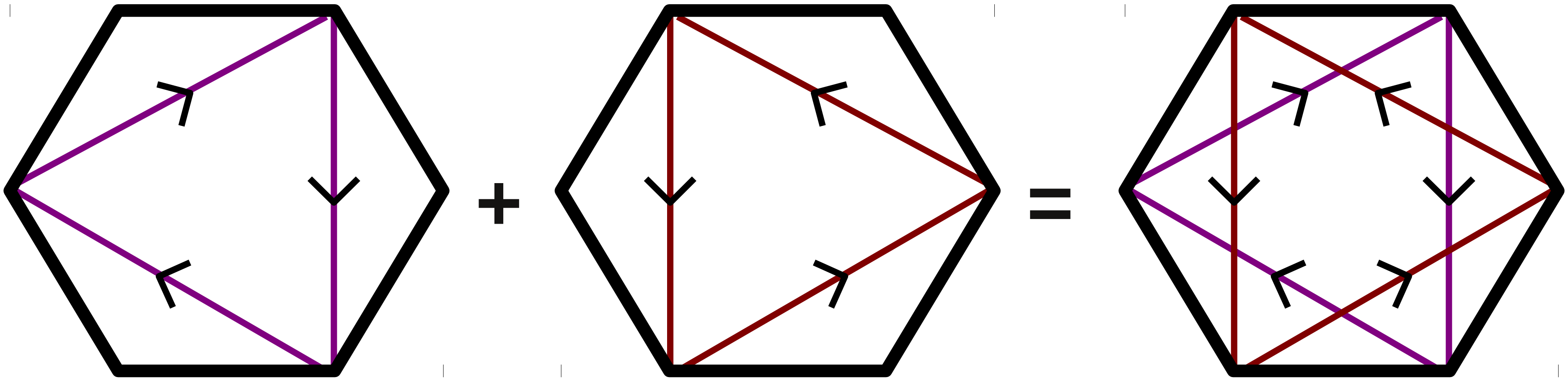

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V_1 \sum_{\langle i,j \rangle} n_i n_j + V_2 \sum_{\langle\langle i,j \rangle\rangle} n_i n_j$$

- *Spinless*: so no Hubbard *U* (& no $\mathcal{T}$– broken temptations)

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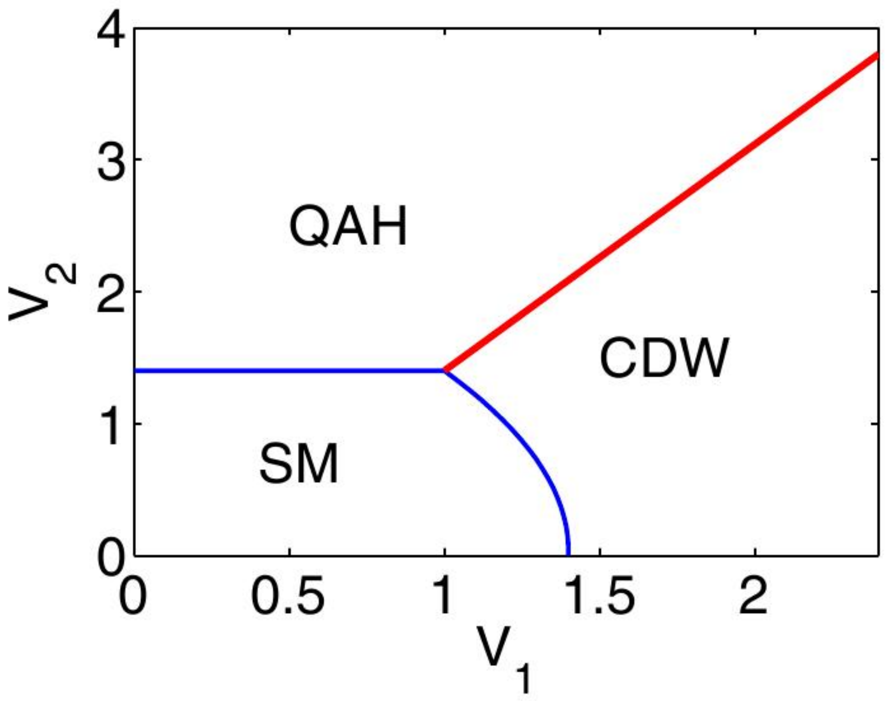
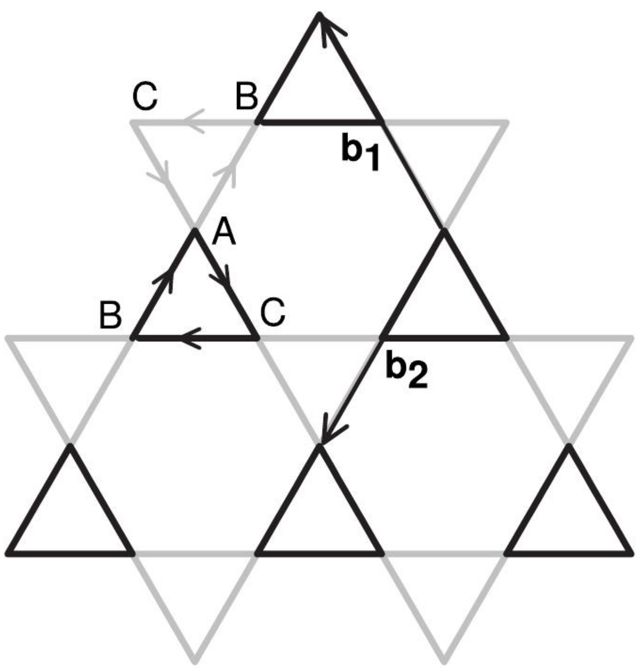
- Mean field
- Half filling

V2 > V1

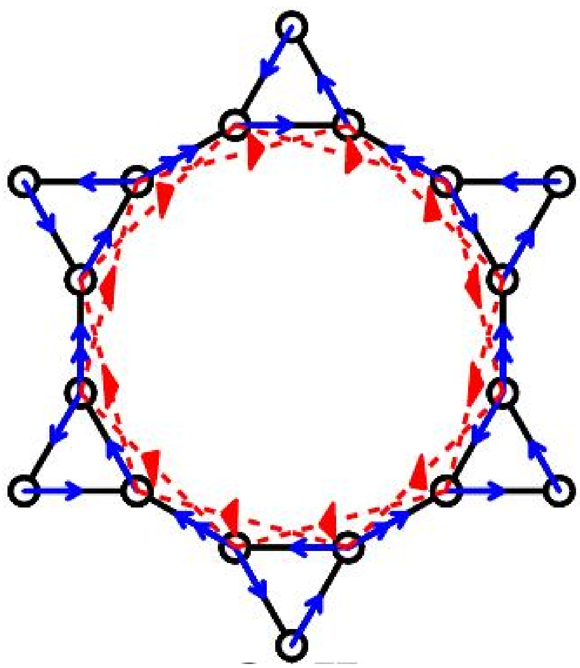


Interactions and "sofisticated" lattices

- *Key:* ***self-consistent orbital currents***
  - Take lattices that naturally allow for intracell, self-consistent currents



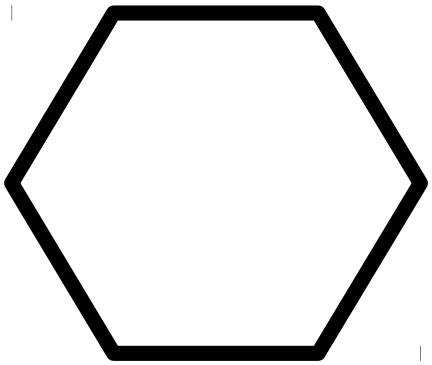
Liu *et al.* PRB, (2010)



Wen *et al.* PRB, (2010)

Interactions and enlarged unit cells

- *Key:* ***self-consistent orbital currents***
  - Just ***enlarge the unit cell*** to allow for intracell, self-consistent currents



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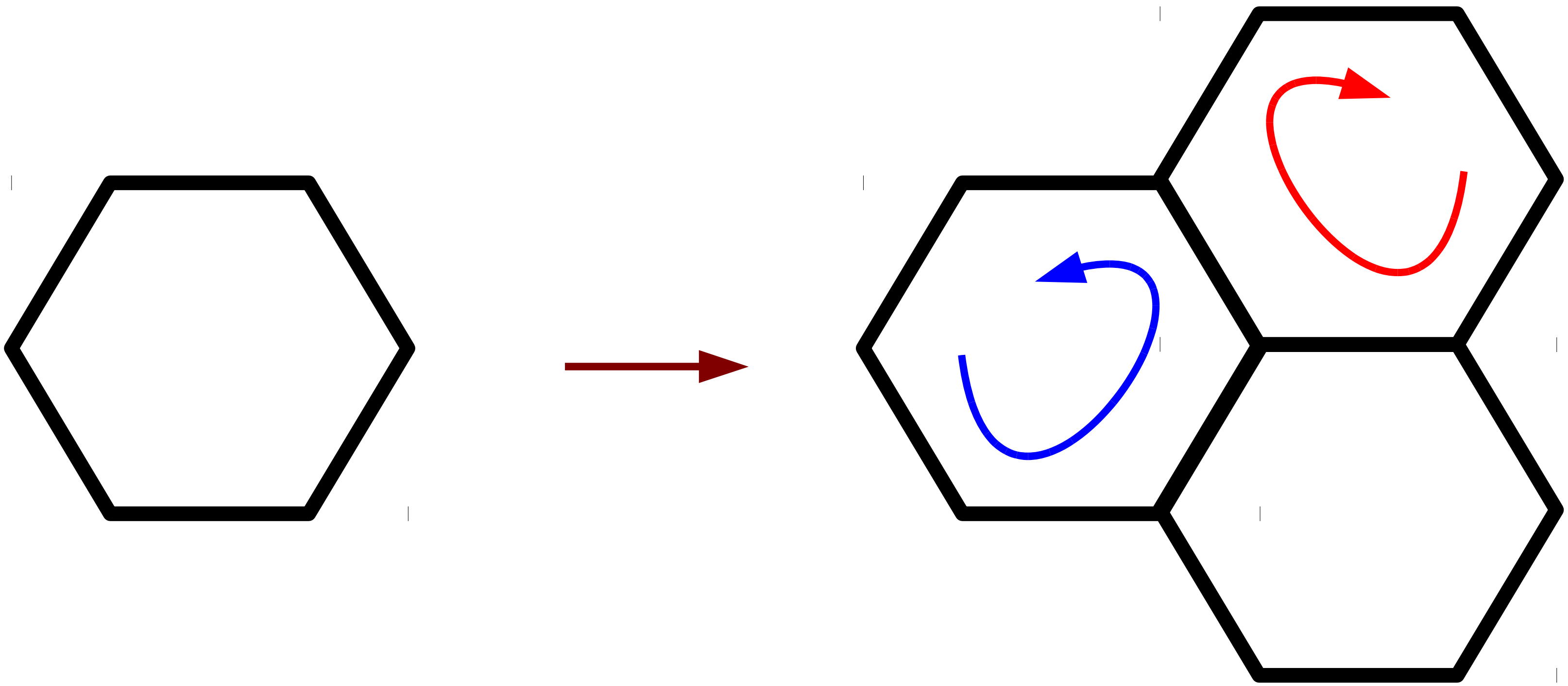

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Case study: the honeycomb lattice

- *Simplest (spinless) Hamiltonian*
  - (For fermions on a lattice with Coulomb interaction)

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_{\langle i,j \rangle} n_i n_j$$

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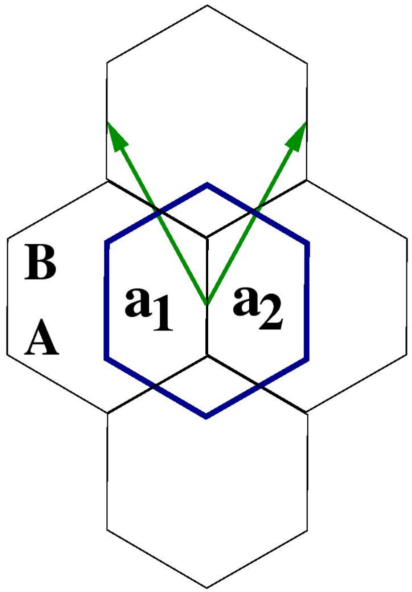
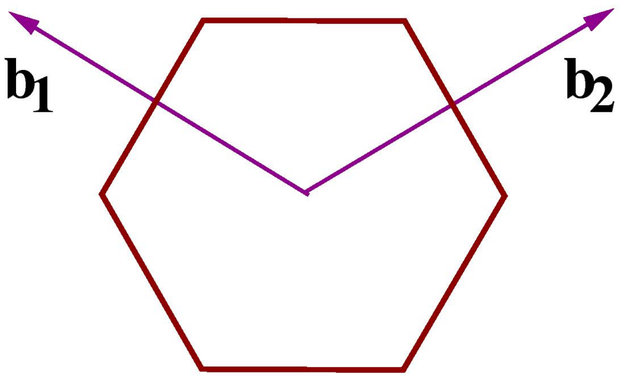
- *From 2 to 6 atom unit cell*





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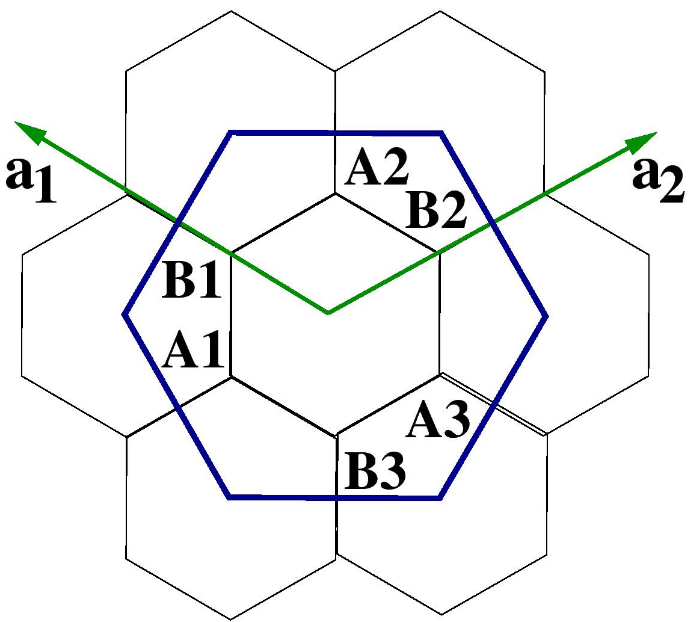
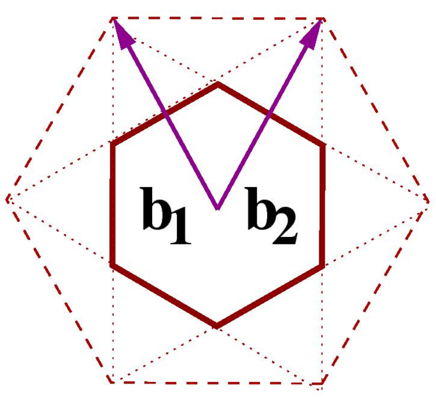




(not dynamical) Mean field

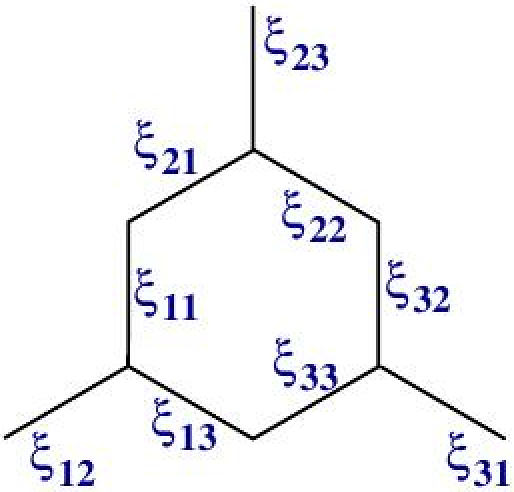
- *Variational mean field method*
  - Bogoliubov – Feynmann – Gibbs inequality

$$\Omega \leq \Omega_{MF} + \langle \mathcal{H} - \mathcal{H}_{MF} \rangle_{MF}$$

$$\mathcal{H} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_{\langle i,j \rangle} n_i n_j$$

$$\mathcal{H}_{MF} = -t \sum_{\langle i,j \rangle} c_i^\dagger c_j + V \sum_{\langle i,j \rangle} \xi_{ij} c_i^\dagger c_j$$

  - *nine complexed valued order parameters:*



Phase diagram

- *Mean field equation + Luttinger's theorem*

$$\xi_{ij} = -\frac{2}{N}\sum_{\mathbf{k}} \gamma_{\mathbf{k}}^{ij} \langle c_j^\dagger(\mathbf{k}) c_i(\mathbf{k}) \rangle_{MF}$$

$$\frac{N_e}{N} = 3 + n = \frac{1}{N}\sum_{\mathbf{k},l} n_l^0(\mathbf{k},\mu)$$

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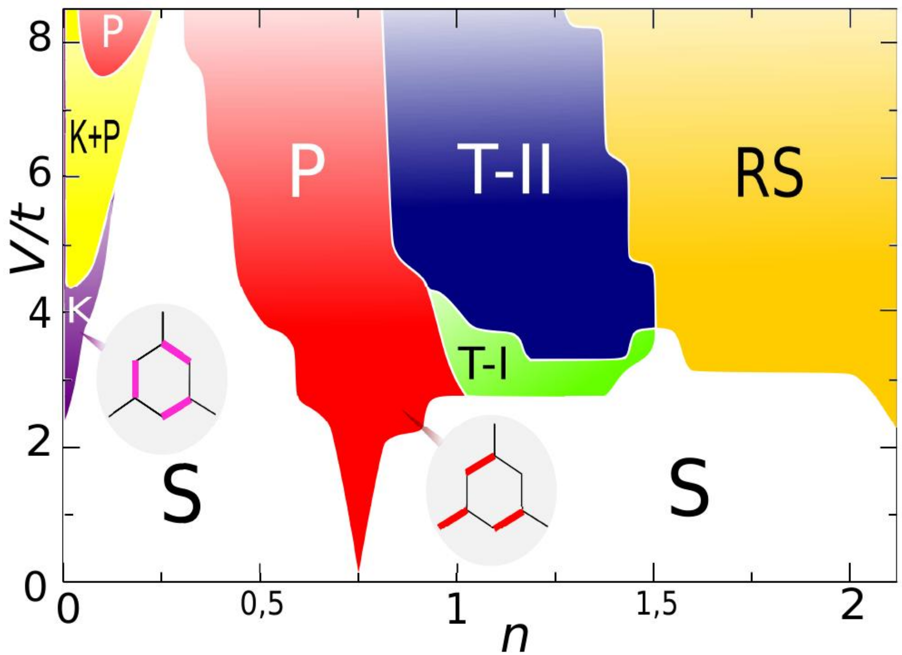
- *n vs V phase diagram:*

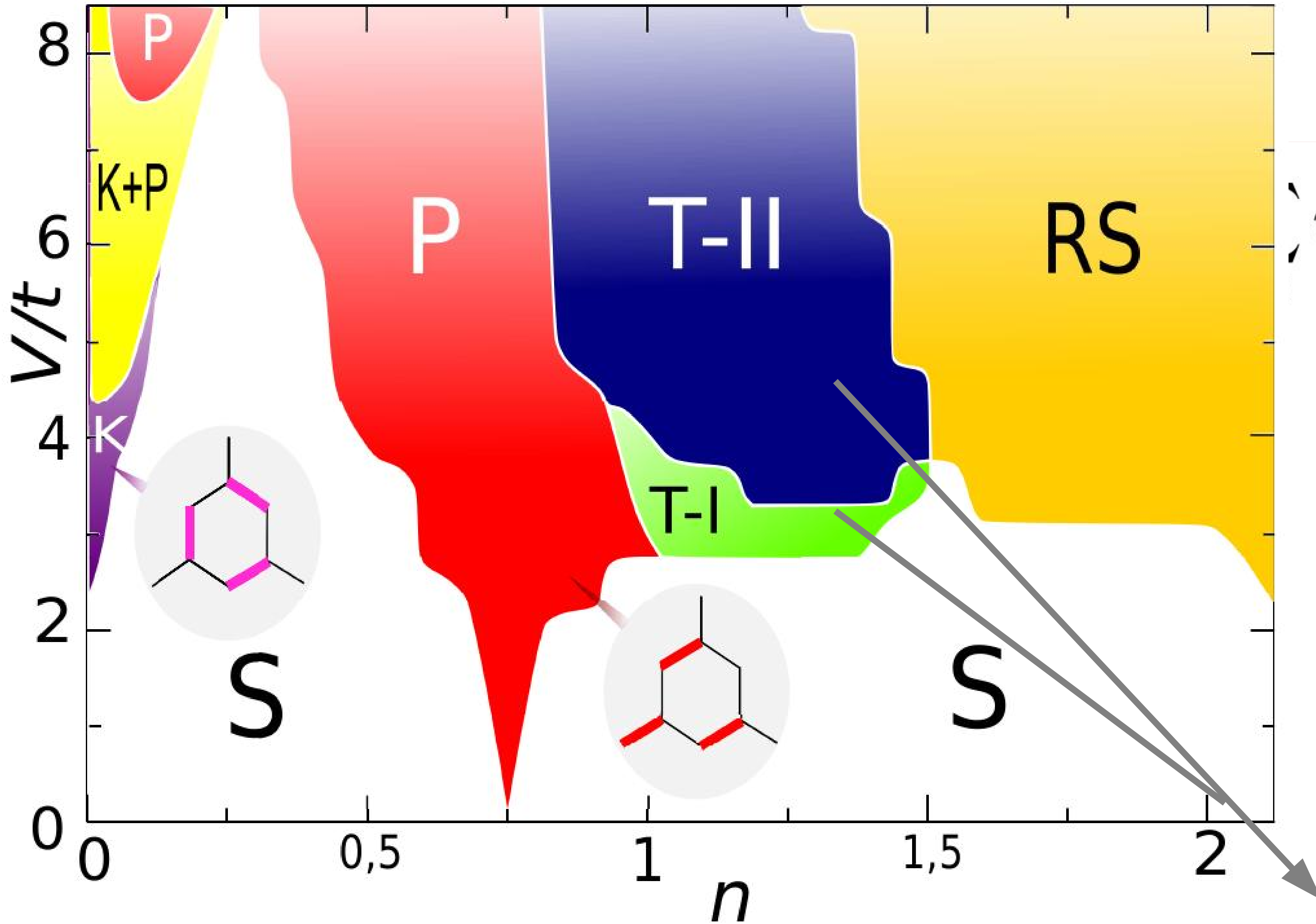


Phase diagram

- *Me*

$\xi_{ij} =$ … $n_l^0(\mathbf{k},\mu)$

- *n v*

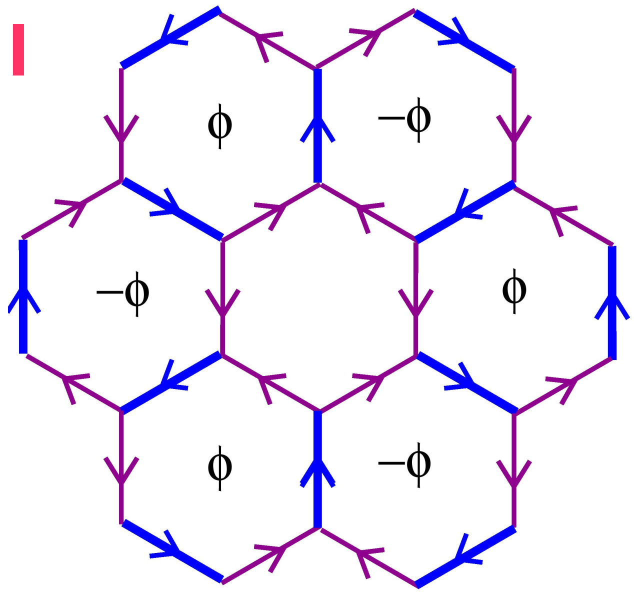


$\mathcal{T}$ – broken phases

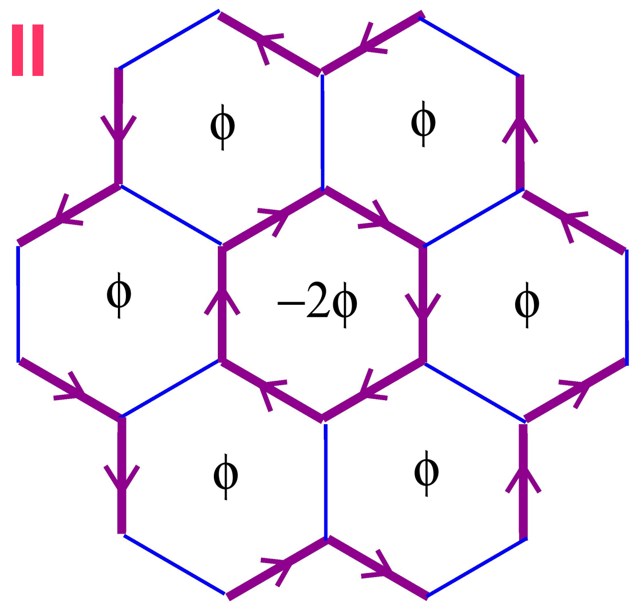
The $\mathcal{T}$ - broken phases

- *Different flux (orbital current) patterns*

$\mathcal{T}$ – I



$\mathcal{T}$ – II



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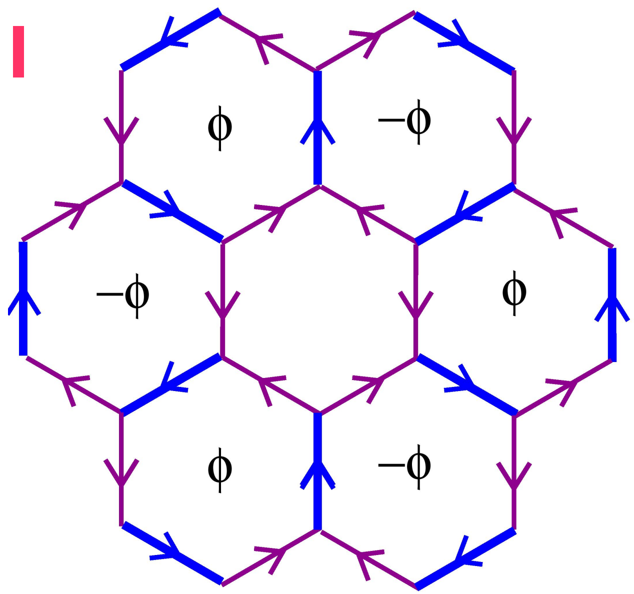
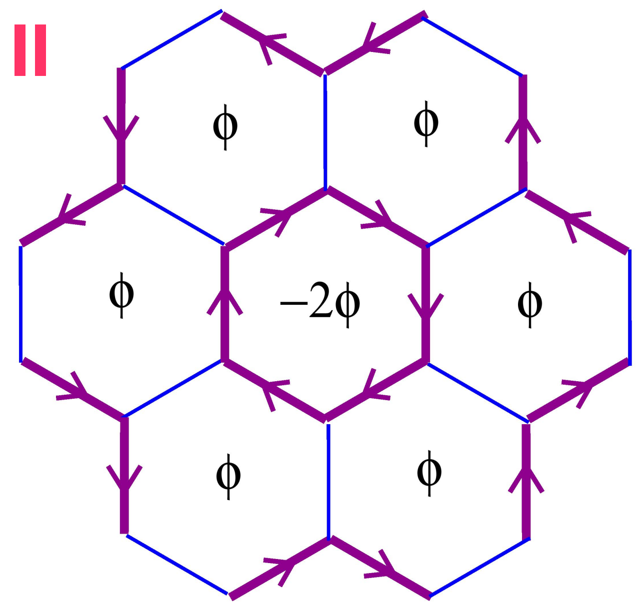


$\mathcal{T}$ I $\mathcal{T}I$

$\mathcal{T}$ – II



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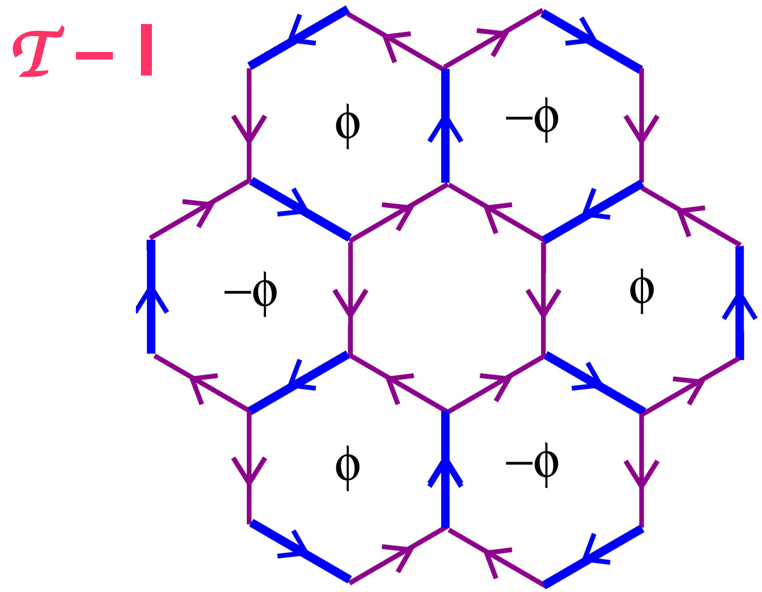

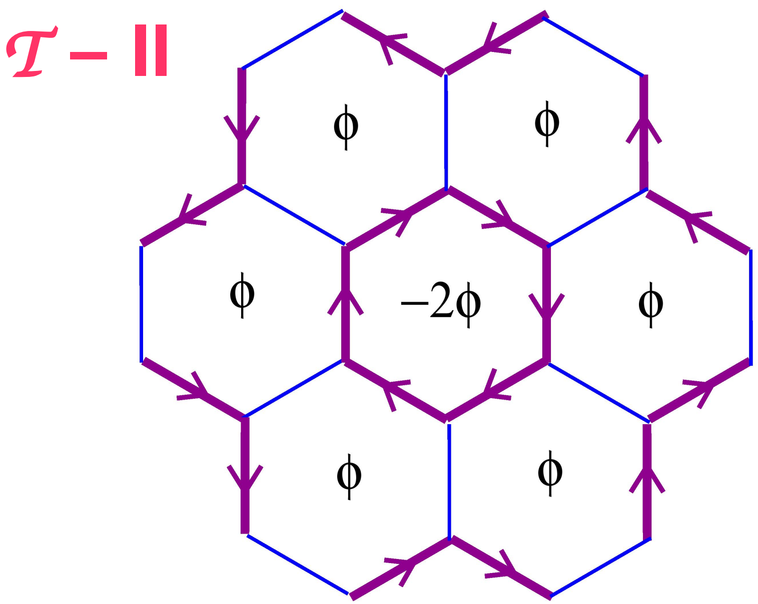

$\not{\mathcal{T}}$ $\quad I \quad \mathcal{T}I$

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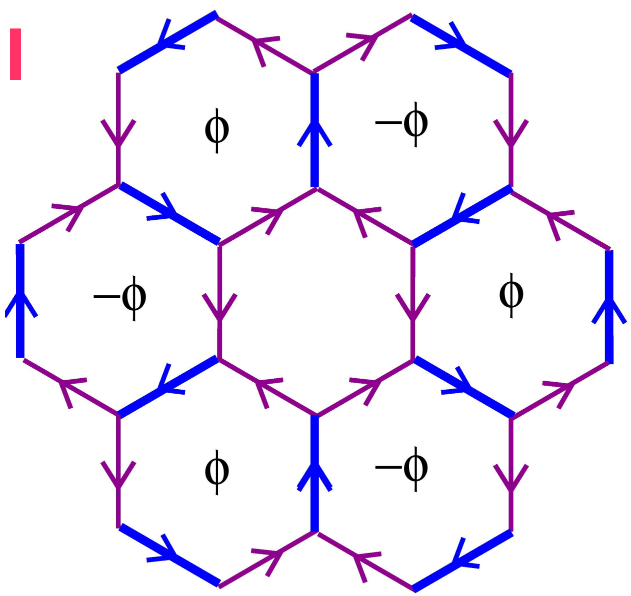
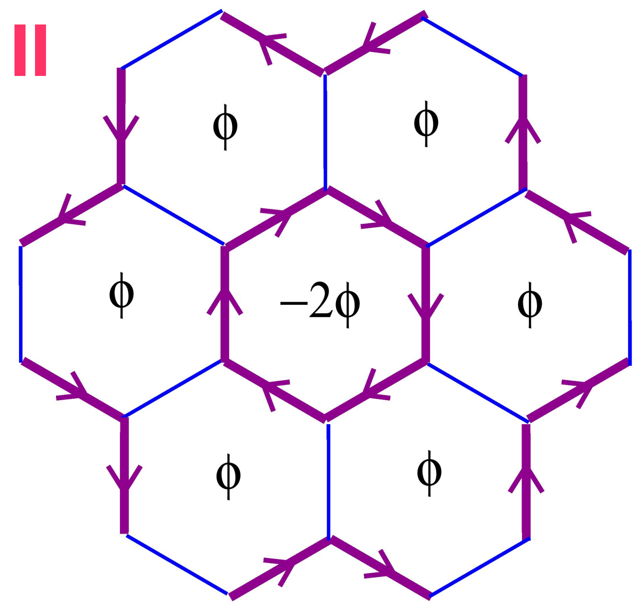


$\not{\mathcal{T}}$ $\not{I}$ $\mathcal{T}I$

$\mathcal{T}$ – II



$\not{\mathcal{T}}$ I $\not{\mathcal{T}I}$

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The $\mathcal{T}$ - broken phases

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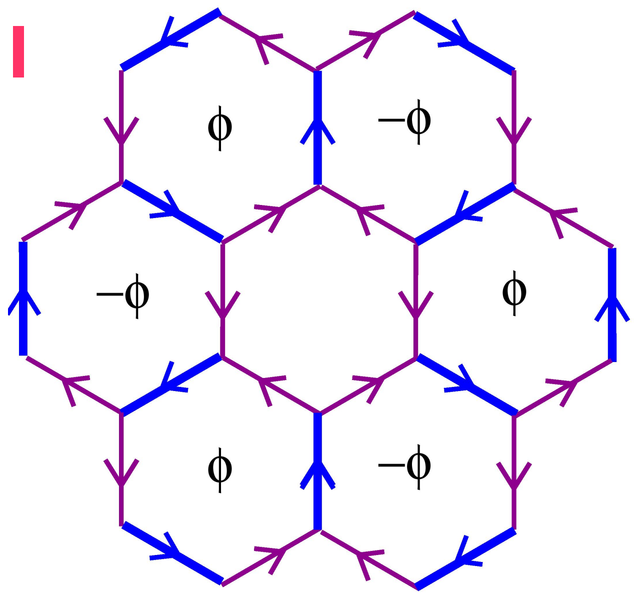

$\not{\mathcal{T}}$ $\not{I}$ $\mathcal{T}I$

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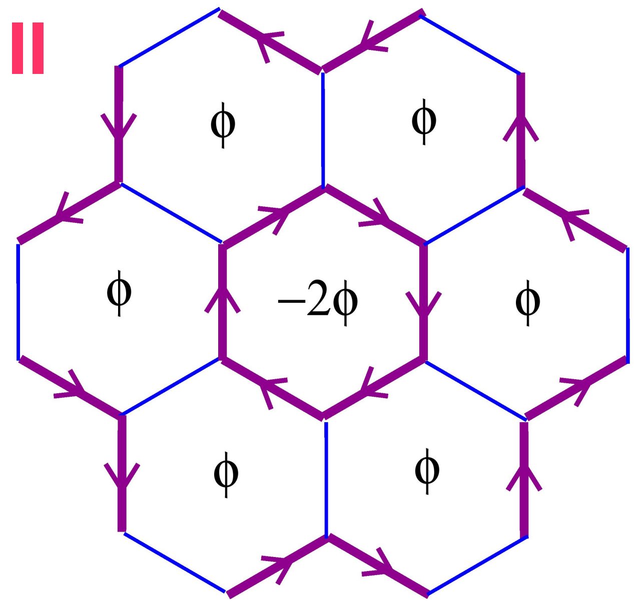


$\not{\mathcal{T}}$ I $\not{\mathcal{T}I}$

$$\sigma^{xy}(\mu) = 0$$

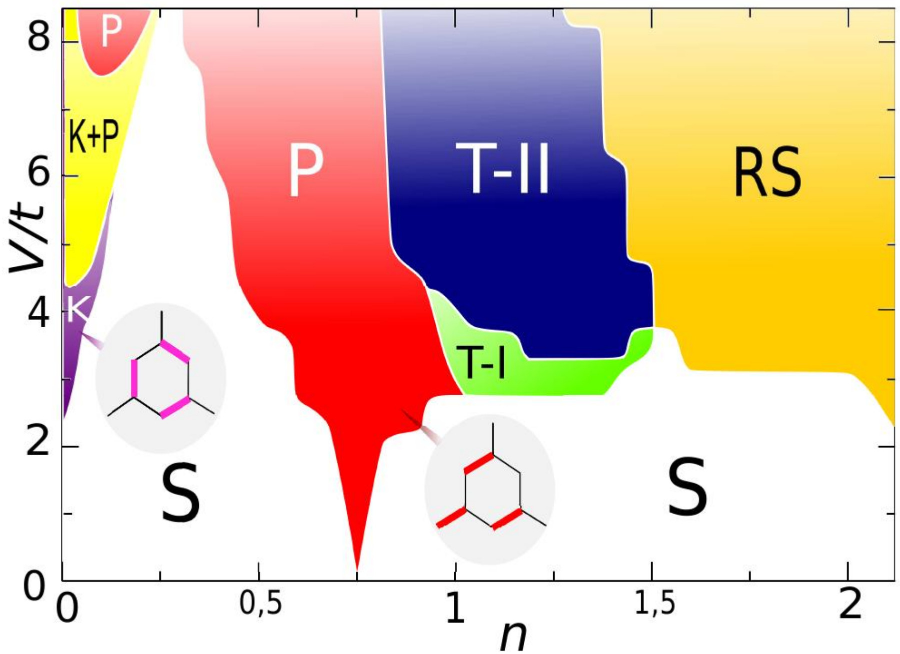
$$\sigma_0^{ab}(\mu) = \frac{e^2}{\hbar}\frac{1}{\Omega N}\sum_{kn}\mathcal{F}_n^{ab} n_n^0(\mathbf{k}, \mu)$$

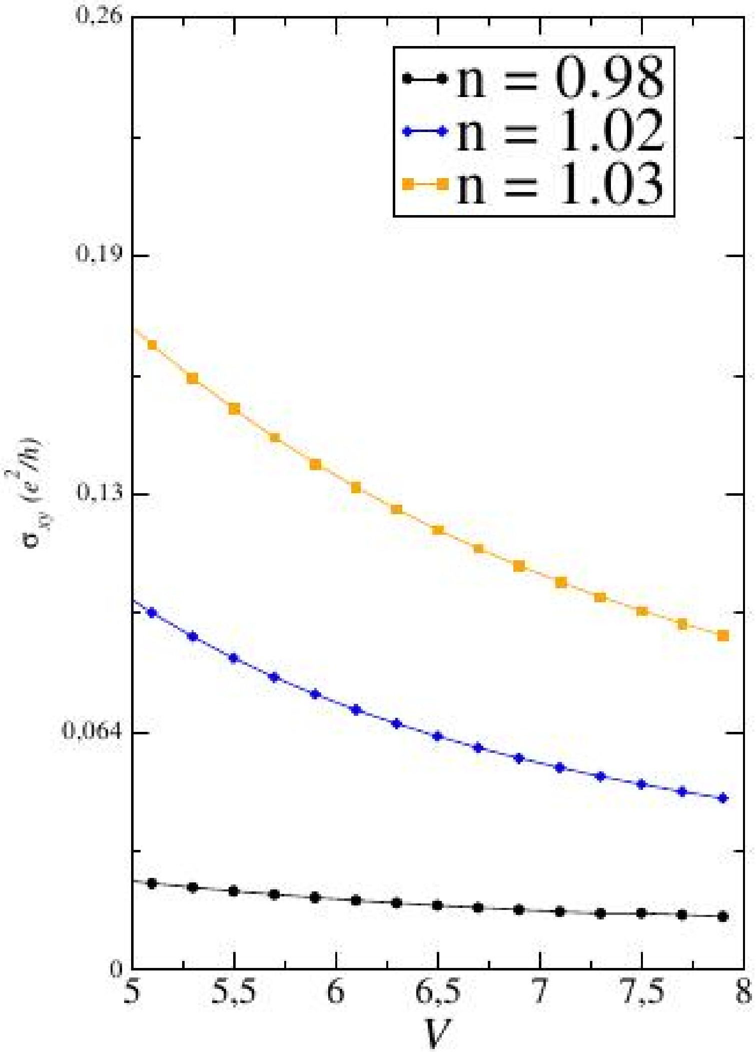
$$\sigma^{xy}(\mu) \neq 0$$

The Anomalous Hall phase

- *Finite AH conductivity in* $\mathcal{T}$ **– II**

$$\sigma_0^{ab}(\mu) = \frac{e^2}{\hbar}\frac{1}{\Omega N}\sum_{kn}\mathcal{F}_n^{ab} n_n^0(\mathbf{k}, \mu)$$





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Conclusions

- *Topologically non-trivial phases via electronic (Coulomb) correlations are possible*

- *Relaxing translational symmetry by enlarging the unit cell enables for T – broken states*
  - Intracell, self-consistent orbital currents possible even in simple lattices
  - Anomalous Hall phases allowed

- *Case study: honeycomb lattice, 6–atom unit cell*
  - Rich phase diagram with T – broken phases
  - Topological Fermi liquid is dominant T – broken state

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